Top-Down Syntax Analysis

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Subjects

- Functionality and Method
- Recursive Descent Parsing
- Using parsing tables
- Explicit stacks
- Creating the table
- LL(k)–grammars
- Other properties
- Handling Limitations

Top-Down Syntax Analysis

input: A sequence of symbols (symbols)

output: A syntax tree or an error message

method

- Read input from left to right
- Construct the syntax tree in a top-down manner starting with a node labeled with the start symbol
- until input accepted (or error) do
  - Predict expansion for the actual leftmost nonterminal (maybe using some lookahead into the remaining input) or
  - Verify predicted terminal symbol against next symbol of the remaining input

Finds leftmost derivations.

Grammar for Arithmetic Expressions

Left factored grammar G_2, i.e. left recursion removed.

$$
\begin{aligned}
S &\rightarrow E \\
E &\rightarrow T E' \\
E' &\rightarrow +E \mid \epsilon \\
T &\rightarrow F T' \\
T' &\rightarrow *T \mid \epsilon \\
F &\rightarrow \textbf{id} \mid (E)
\end{aligned}
$$

Recursive Descent Parsing

- parser is a program,
- a procedure X for each non-terminal X,
  - procedure X parses words for non-terminal X,
  - starts with the first symbol read (into variable $nextsym$),
  - ends with the following symbol read (into variable $nextsym$).
- uses one symbol lookahead into the remaining input.
- uses the **FiFo** sets to make the expansion transitions deterministic

$$\textbf{FiFo}(N \rightarrow \alpha) = FIRST_1(\alpha) \oplus_1 FOLLOW_1(N)$$

$$= \begin{cases} FIRST_1(\alpha) \cup FOLLOW_1(N) & \alpha \stackrel{*}{\Longrightarrow} \epsilon \\ FIRST_1(\alpha) & \text{otherwise} \end{cases}$$

Parser for G_2

```
program parser;
var  nextsym: string;
proc  scan;
 {reads next input symbol into  nextsym}

proc  error (message: string);
 {issues error message and stops parser}

proc  accept; {terminates successfully}

proc  S;
 begin  E end ;

proc  E;
 begin T; E' end ;

proc  E';
 begin
   case  nextsym in
    {"+"} :if  nextsym = "+"
             then  scan
             else  error( "+" expected) fi ; E;
    otherwise ;
```

```
    endcase
  end ;
proc  T;
  begin F; T' end ;

proc  T';
  begin
    case  nextsym in
     {" * "} :if  nextsym = "*"
               then  scan
               else  error( "*" expected) fi ; T;
     otherwise ;
    endcase
  end ;
proc  F;
  begin
    case  nextsym in
     {"("} : if  nextsym = "("
               then  scan
               else  error( "(" expected) fi ; E;
               if nextsym = ")"
               then scan else error(" ) expected") fi;
     otherwise if nextsym ="id"
```

```
              then scan else error("id expected") fi;
   endcase
 end ;
begin
scan; S;
if  nextsym = "#" then  accept else  error("# expected") fi
end .
```

Parser Schema

```
program parser;
    var nextsym: symbol;
    proc scan;
        (* reads next input symbol into nextsym *)
    proc error (message: string);
        (* issues error message and stops the parser *)
    proc accept;
        (* terminates parser successfully *)
    N_prog(X_0);                    (* X_0 start symbol *)
    N_prog(X_1);
        ⋮
    N_prog(X_n);
begin
    scan;
    X_0;
    if nextsym = "#"
        then accept
        else error(". . .")
    fi
end
```

The Non-terminal Procedures

N = Non-terminal, C = Concatenation, S = Symbol

N_prog(X) = (* $X \rightarrow \alpha_1 | \alpha_2 | \cdots | \alpha_{k-1} | \alpha_k$ *)

```
        proc  X;
        begin
        case  nextsym in
        FiFo(α1) :     C_progr(α1);
        FiFo(α2) :     C_progr(α2);
              ⋮
        FiFo(αk−1) : C_progr(αk−1);
        otherwise C_progr(αk);
        endcase
        end ;
```

C_progr($\alpha_1 \alpha_2 \cdots \alpha_k$) =

S_progr(α_1); S_progr(α_2); ... S_progr(α_k);

S_progr(a) =

```
        if  nextsym = a  then  scan
        else  error ( "a expected" )
        fi
```

S_progr(Y) = Y

FiFo–sets should be disjoint (LL(1)–grammar)

A Generative Solution

- At compiler–generation time construct a table M
  $M : V_N \times V_T \to P$
  $M[N, a]$ is the production used to expand nonterminal N when the current symbol is a

- For some grammars report that the table cannot be constructed
  The compiler writer can then decide to:
  - change the grammar (but not the language)
  - use a more general parser-generator
  - "Patch" the table (manually or using some rules)

Creating the table

Input: cfg G, $FIRST_1$ und $FOLLOW_1$ for G.

Output: The parsing table M or an indication that such a table cannot be constructed

Method: M is constructed as follows:
For all $X \rightarrow \alpha \in P$ and $a \in FIRST_1(\alpha)$,
set $M[X, a] = (X \rightarrow \alpha)$.
If $\varepsilon \in FIRST_1(\alpha)$, for all $b \in FOLLOW_1(X)$
set $M[X, b] = (X \rightarrow \alpha)$.
Set all other entries of M to $error$.

Parser table cannot be constructed if at least one entry is set twice.
G is not LL(1)

Example – arithmetic expressions

nonterminal	symbol	Production
S	$(, id$	$S \rightarrow E$
S	$+, *,), \#$	$error$
E	$(, id$	$E \rightarrow TE'$
E	$+, *,), \#$	$error$
E'	$+$	$E' \rightarrow +E$
E'	$), \#$	$E' \rightarrow \epsilon$
E'	$(, *, id$	$error$
T	$(, id$	$T \rightarrow FT'$
T	$+, *,), \#$	$error$
T'	$*$	$T' \rightarrow *T$
T'	$+,), \#$	$T' \rightarrow \epsilon$
T'	$(, id$	$error$
F	id	$F \rightarrow id$
F	$($	$F \rightarrow (E)$
F	$+, *,)$	$error$

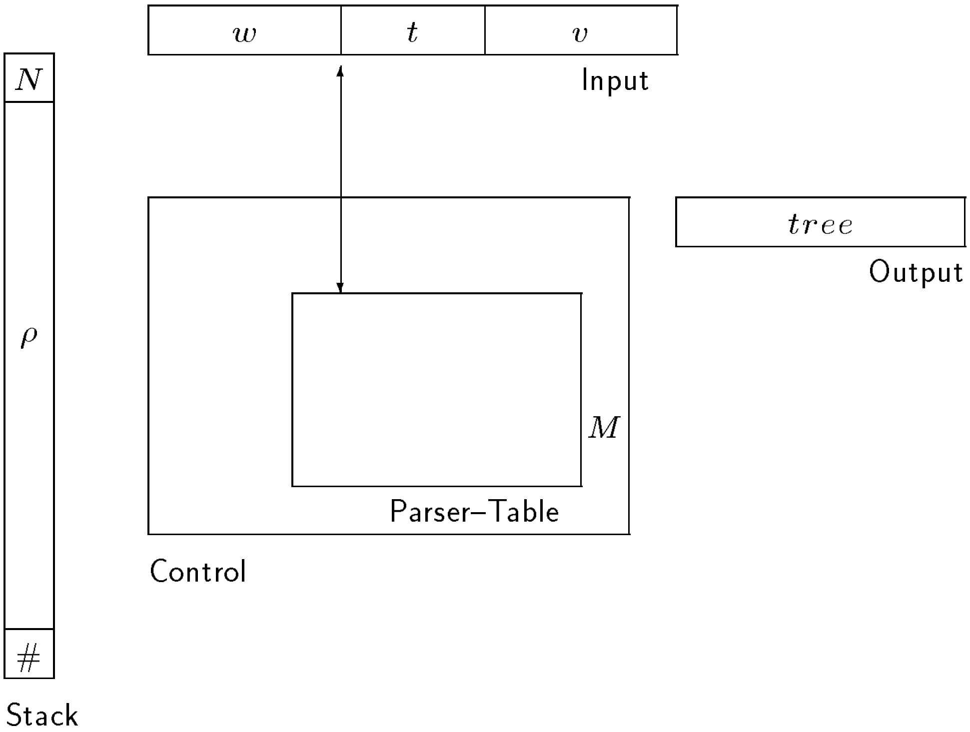
LL-Parser Driver

Interprets the table M

```
program parser;
    var nextsym: symbol;
    var st: stack of item;
    proc scan;
        (* reads next input symbol into nextsym *)
    proc error (message: string);
        (* issues error message and stops the parser *)
    proc accept;
        (* terminates parser successfully *)
    proc reduce;
        (* replaces [X → β.Yγ][Y → α.]
        by [X → βY.γ] *)
    proc pop;
        (* removes topmost item from st *)
    proc push ( i : item);
        (* pushes i onto st *)
    proc replaceby ( i: item);
        (* replaces topmost item of st by i *)
```

```
begin
    scan; push( [S' → .S] );
    while  nextsym ≠ "#" do
    case top in
    [X → β.aγ]:      if  nextsym = a
                     then  scan; replaceby([X → βa.γ]
                     else error fi ;
    [X → β.Yγ] :     if  M[Y, nextsym] = (Y → α)
                     then  push([Y → .α])
                     else  error fi ;
    [X → α.]:        reduce;
    [S' → S.] :      if  nextsym = "#" then  accept
                     else  error fi
    endcase
    od
end  .
```

Explicit Stack
Deterministic Pushdown Automaton



LL(k)-grammar

Let $G = (V_N, V_T, P, S)$ be a cfg and k be a natural number.
G is an **LL(k)-grammar** iff the following holds:
if there exist two leftmost derivations
$S \underset{lm}{\overset{*}{\Longrightarrow}} uY\alpha \underset{lm}{\Longrightarrow} u\beta\alpha \underset{lm}{\overset{*}{\Longrightarrow}} ux$ and
$S \underset{lm}{\overset{*}{\Longrightarrow}} uY\alpha \underset{lm}{\Longrightarrow} u\gamma\alpha \underset{lm}{\overset{*}{\Longrightarrow}} uy$, and if $k : x = k : y$,
then $\beta = \gamma$.

The expansion of the leftmost non-terminal is always uniquely determined by

- the consumed part of the input and
- the next k symbols of the remaining input

Example1

Let G_1 the cfg with the productions

$$
\begin{array}{lcll}
STAT & \rightarrow & \textbf{if id then } STAT \textbf{ else } STAT \textbf{ fi} & | \\
 & & \textbf{while id do } STAT \textbf{ od} & | \\
 & & \textbf{begin } STAT \textbf{ end} & | \\
 & & \textbf{id} := \textbf{id} &
\end{array}
$$

G_1 is an LL(1)-grammar.

$$
\begin{array}{lclclcl}
STAT & \overset{*}{\underset{lm}{\Longrightarrow}} & w \; STAT \, \alpha & \underset{lm}{\Longrightarrow} & w \, \beta \, \alpha & \overset{*}{\underset{lm}{\Longrightarrow}} & w \, x \\
STAT & \overset{*}{\underset{lm}{\Longrightarrow}} & w \; STAT \, \alpha & \underset{lm}{\Longrightarrow} & w \, \gamma \, \alpha & \overset{*}{\underset{lm}{\Longrightarrow}} & w \, y
\end{array}
$$

From $1 : x = 1 : y$ follows $\beta = \gamma$,
e.g., from $1 : x = 1 : y = \textbf{if}$
follows $\beta = \gamma = \textbf{if id then } STAT \textbf{ else } STAT \textbf{ fi}$

Example2

Let G_2 be the cfg with the productions

$$
\begin{array}{lll}
STAT & \rightarrow & \textbf{if}\ \ \textbf{id then}\ STAT\ \textbf{else}\ STAT\ \textbf{fi} \quad | \\
 & & \textbf{while}\ \ \textbf{id do}\ STAT\ \textbf{od} \quad | \\
 & & \textbf{begin}\ STAT\ \textbf{end} \quad | \\
 & & \textbf{id} := \textbf{id} \quad | \\
 & & \textbf{id}:\ STAT \quad | \qquad (*\ \text{labeled statem.}\ *) \\
 & & \textbf{id}\ (\textbf{id}\ ) \qquad (*\ \text{procedure call}\ *)
\end{array}
$$

G_2 is not an LL(1)–grammar.

$$STAT \overset{*}{\underset{lm}{\Longrightarrow}} w\ STAT\ \alpha \underset{lm}{\Longrightarrow} w\ \overbrace{\textbf{id} := \textbf{id}}^{\beta}\ \alpha \overset{*}{\underset{lm}{\Longrightarrow}} w\,x$$

$$STAT \overset{*}{\underset{lm}{\Longrightarrow}} w\ STAT\ \alpha \underset{lm}{\Longrightarrow} w\ \overbrace{\textbf{id} : STAT}^{\gamma}\ \alpha \overset{*}{\underset{lm}{\Longrightarrow}} w\,y$$

$$STAT \overset{*}{\underset{lm}{\Longrightarrow}} w\ STAT\ \alpha \underset{lm}{\Longrightarrow} w\ \overbrace{\textbf{id}(\textbf{id})}^{\delta}\ \alpha \overset{*}{\underset{lm}{\Longrightarrow}} w\,z$$

and $1 : x = 1 : y = 1 : z =$ "**id**",
and β, γ, δ are pairwise different.

G_2 is an LL(2)–grammar.
$2 : x =$ "**id** :=", $2 : y =$ "**id** :", $2 : z =$ "**id**(" are pairwise different.

Example3

Let G_3 have the productions

$$
\begin{array}{lcll}
STAT & \rightarrow & \textbf{if } \ \textbf{id then } \ STAT \ \textbf{else} \ \ STAT \ \textbf{fi} & | \\
 & & \textbf{while } \ \textbf{id do } STAT \ \textbf{od} & | \\
 & & \textbf{begin } \ STAT \ \textbf{end} & | \\
 & & VAR := \ VAR & | \\
 & & \textbf{id}(\ IDLIST\ ) \qquad (* \text{ procedure call } *) & \\
VAR & \rightarrow & \textbf{id} \ \ | \ \ \textbf{id}\,(IDLIST) \ (* \text{ indexed variable } *) & \\
IDLIST & \rightarrow & \textbf{id} \ \ | \ \ \textbf{id},\ IDLIST &
\end{array}
$$

G_3 is not an LL(k)–grammar for any k.

Proof: Assume G_3 to be LL(k) for a $k > 0$.

Let $STAT \Rightarrow \beta \overset{*}{\underset{lm}{\Longrightarrow}} x$ and $STAT \Rightarrow \gamma \overset{*}{\underset{lm}{\Longrightarrow}} y$ with
$x = \textbf{id}\,(\underbrace{\textbf{id}, \textbf{id}, \ldots, \textbf{id}}_{\lceil \frac{k}{2} \rceil \text{ times}}) := \textbf{id}$ and $y = \textbf{id}\,(\underbrace{\textbf{id}, \textbf{id}, \ldots, \textbf{id}}_{\lceil \frac{k}{2} \rceil \text{ times}})$

Then $k : x = k : y$,
but $\beta = VAR := VAR \neq \gamma = \textbf{id}\,(IDLIST)$.

Transforming to LL(k)

Factorization creates an LL(2)–grammar, equivalent to G_3.

The productions

$$STAT \quad \rightarrow \quad VAR := VAR \quad | \quad \textbf{id}(IDLIST)$$

are replaced by

$$\begin{array}{lcl} STAT & \rightarrow & ASSPROC \quad | \quad \textbf{id} := VAR \\ ASSPROC & \rightarrow & \textbf{id}(IDLIST)\ APREST \\ APREST & \rightarrow & := VAR \quad | \quad \varepsilon \end{array}$$

A non–LL(k)–language

Let $G_4 = (\{S, A, B\}, \{0, 1, a, b\}, P_4, S)$

$$P_4 = \left\{ \begin{array}{lcl} S & \rightarrow & A \mid B \\ A & \rightarrow & aAb \mid 0 \\ B & \rightarrow & aBbb \mid 1 \end{array} \right\}$$

$L(G_4) = \{a^n 0 b^n \mid n \geq 0\} \cup \{a^n 1 b^{2n} \mid n \geq 0\}$.

G_4 is not LL(k) for any k.

Consider the two leftmost derivations

$$S \underset{lm}{\overset{0}{\Longrightarrow}} S \underset{lm}{\Longrightarrow} A \underset{lm}{\overset{*}{\Longrightarrow}} a^k 0 b^k$$
$$S \underset{lm}{\overset{0}{\Longrightarrow}} S \underset{lm}{\Longrightarrow} B \underset{lm}{\overset{*}{\Longrightarrow}} a^k 1 b^{2k}$$

With $u = \alpha = \varepsilon$, $\beta = A$, $\gamma = B$, $x = a^k 0 b^k$, $y = a^k 1 b^{2k}$ it holds $k : x = k : y$, but $\beta \neq \gamma$.
Since k can be chosen arbitrarily,
we have G_4 is not LL(k) for any k.
There even is no LL(k)-grammar for $L(G_4)$ for any k.

Towards Checkable LL(k)–conditions

Theorem. *G is LL(k)–grammar iff the following condition holds:*
Are $A \rightarrow \beta$ and $A \rightarrow \gamma$ different productions in P, then

$$FIRST_k(\beta\alpha) \cap FIRST_k(\gamma\alpha) = \emptyset \text{ for all } \alpha \text{ with } S \overset{*}{\underset{lm}{\Longrightarrow}} wA\alpha$$

Theorem. *Let G be a cfg without productions of the form $X \rightarrow \varepsilon$.*
G is an LL(1)–grammar iff for each non-terminal X with the alternatives $X \rightarrow \alpha_1 | \ldots | \alpha_n$ the sets $FIRST_1(\alpha_1), \ldots, FIRST_1(\alpha_n)$ are pairwise disjoint.

Theorem. *G is LL(1) iff*
For different productions $A \rightarrow \beta$ and $A \rightarrow \gamma$
$FIRST_1(\beta) \oplus_1 FOLLOW_1(A) \cap FIRST_1(\gamma) \oplus_1 FOLLOW_1(A) = \emptyset$.

Corollary.
G is LL(1) iff for all alternatives $A \rightarrow \alpha_1 | \ldots | \alpha_n$:

1. $FIRST_1(\alpha_1), \ldots, FIRST_1(\alpha_n)$ are pairwise disjoint; in particular, at most one of them may contain ε

2. $\alpha_i \stackrel{*}{\Longrightarrow} \varepsilon$ implies:

   $FIRST_1(\alpha_j) \cap FOLLOW_1(A) = \emptyset$ for $1 \leq j \leq n,\ j \neq i$.

Further Definitions and Theorems

- G is called a **strong LL(k)-grammar** if for each two different productions $A \rightarrow \beta$ and $A \rightarrow \gamma$

  $$FIRST_k(\beta) \oplus_k FOLLOW_k(A) \cap FIRST_k(\gamma) \oplus_k FOLLOW_k(A) = \emptyset,$$

- A production is called **directly left recursive**, if it has the form $A \rightarrow A\alpha$

- A non-terminal A is called **left recursive** if it has a derivation $A \stackrel{+}{\Longrightarrow} A\alpha$.

- A cfg G is called **left recursive**, if G contains at least one left recursive non-terminal

Theorem.

(a) *G is not LL(k) for any k if G is left recursive.*

(b) *G is not ambiguous if G is LL(k)-grammar.*