# Compiling Simple Assignments

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# Subjects

- Compile- vs. Run-Time Information
- L-values vs. R-values
- The P-machine
- Code generation for expressions and assignments

# Compile- vs. Run-Time Information

**compile-time-information (static):** Information contained in or derivable from the source program

- The source code
- The types of variables (in most languages)
- The scope of variables
- The values of constants
- The addresses of static variables
- The relative addresses of automatic variables
- The size of static data (scalars, stat. arrays, records)
- ⋮

**run-time-information (dynamic):** Information only available at run time

- The values of variables
- The values of conditions
- The depth of recursion
- The size of dynamic data (dyn. arrays, lists, trees)
- ⋮

# L-values vs. R-values

- Assignment $x := exp$ is compiled into:
  1. Compute the **address** of $x$
  2. Compute the **value** of $exp$
  3. Store the value of $exp$ at the address of $x$
- Generalization

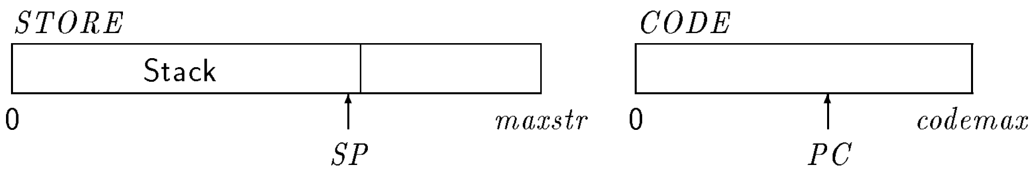
  **R-value**

$$
\begin{aligned}
\text{r-val}(x) &= \text{value of } x \\
\text{r-val}(5) &= 5 \\
\text{r-val}(x+y) &= \text{r-val}(x) + \text{r-val}(y)
\end{aligned}
$$

  **L-value**

$$
\begin{aligned}
\text{l-val}(x) &= \text{address of } x \\
\text{l-val}(5) &= \textbf{undefined} \\
\text{l-val}(x+y) &= \textbf{undefined} \\
\text{l-val}(a[i]) &= \text{l-val}(a) + \text{some function of r-val}(i)
\end{aligned}
$$

# The P-Machine




- Memory
- Instructions operate on the stack
- Typed instructions (using Pascal ordinal types)
  $+_i$
  1. adds the two top most int values on the stack;
  2. removes these from the stack;
  3. stores (pushes) the result on the stack
- Operand types in the P-machine

| | |
|---|---|
| i | integer |
| r | real |
| a | address |
| b | boolean |

- Type indications in the instruction definitions

| | |
|---|---|
| T | all types |
| N | numerical types |

# P-machine main loop

**while true do begin**
$PC := PC + 1$ ;
execute instruction in location $CODE[PC - 1]$
**end**;

Why PC-increment before instruction execution?
Reason: jumps and procedure calls

# Example Program

**Pascal-program**

**program** foo(input, output) ;
**var** x, y : **integer**;
**begin**
    x := 3 ;
    y := x + 7
**end.**

**P-code-program**

| | | | |
|---|---|---|---|
| p | | initialization code | |
| **ssp** | 8 | | allocate stack space |
| **sep** | 3 | | space for intermediate computations |
| **ldc** | a | 5 | pushes the address of x |
| **ldc** | i | 3 | pushes 3 |
| **sto** | i | | stores 3 in x |
| **ldc** | a | 6 | pushes the address of y |
| **ldo** | i | 5 | pushes the value of x |
| **ldc** | i | 7 | pushes 7 |
| **add** | i | | pushes x + 7 |
| **sto** | i | | stores x+7 in y |
| **stp** | | | |

# The definition of P-instructions

| Instruction | Meaning | Condition | Result |
|---|---|---|---|
| | | | |

**Instruction:** name of the instruction and list of its parameters,

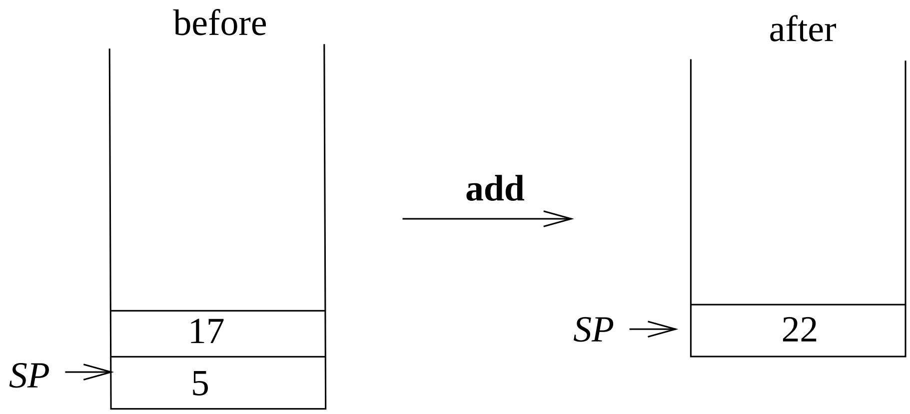
**Meaning:** program in a very reduced imperative language consisting of assignments between machine ressources and conditionals,

**Condition:** condition on the execution of the instruction, often a pattern describing the expected contents of the top end of the stack, i.e., the types of the cell contents,

**Result:** description of the result, a pattern describing the resulting stack contents.

# P-instructions for Arithmetic

| Instr. | Meaning | Cond. | Res. |
|---|---|---|---|
| **add** $N$ | $STORE[SP-1] := STORE[SP-1] +_N STORE[SP]$ ; $SP := SP-1$ | $\binom{N}{N}$ | $(N)$ |
| **sub** $N$ | $STORE[SP-1] := STORE[SP-1] -_N STORE[SP]$ ; $SP := SP-1$ | $\binom{N}{N}$ | $(N)$ |
| **mul** $N$ | $STORE[SP-1] := STORE[SP-1] *_N STORE[SP]$ ; $SP := SP-1$ | $\binom{N}{N}$ | $(N)$ |
| **div** $N$ | $STORE[SP-1] := STORE[SP-1] /_N STORE[SP]$ ; $SP := SP-1$ | $\binom{N}{N}$ | $(N)$ |
| **neg** $N$ | $STORE[SP] := -_N STORE[SP]$ | $(N)$ | $(N)$ |



# P-instructions for Boolean Operations

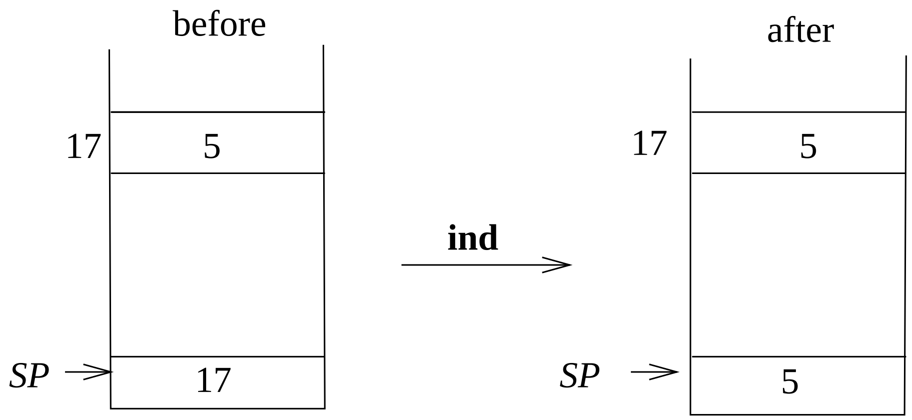
| Instr. | Meaning | Cond. | Res. |
|---|---|---|---|
| **and** | $STORE[SP-1] := STORE[SP-1]\ \mathit{and}\ STORE[SP]\ ;$ <br> $SP := SP-1$ | $\begin{pmatrix} b \\ b \end{pmatrix}$ | $(b)$ |
| **or** | $STORE[SP-1] := STORE[SP-1]\ \mathit{or}\ STORE[SP]\ ;$ <br> $SP := SP-1$ | $\begin{pmatrix} b \\ b \end{pmatrix}$ | $(b)$ |
| **not** | $STORE[SP] := \mathit{not}\ STORE[SP]$ | $(b)$ | $(b)$ |

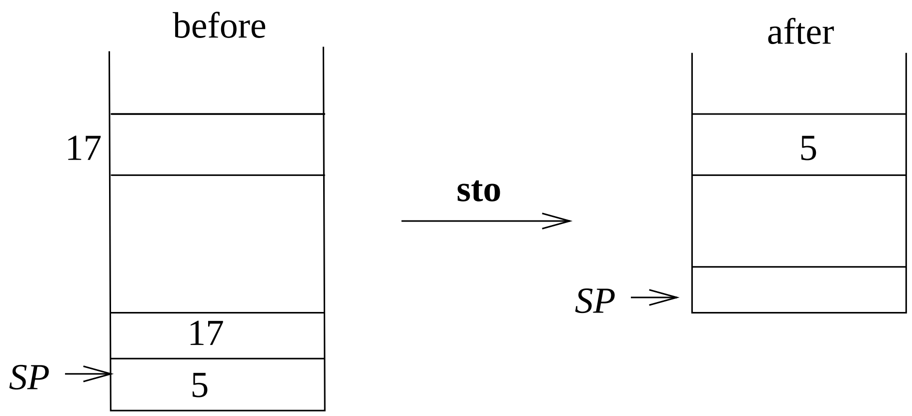
# P-instructions for comparisons

| Instr. | Meaning | Cond. | Res. |
|---|---|---|---|
| **equ** $T$ | $STORE[SP-1] := STORE[SP-1] =_T STORE[SP]$ ; $SP := SP-1$ | $\binom{T}{T}$ | $(b)$ |
| **geq** $T$ | $STORE[SP-1] := STORE[SP-1] \geq_T STORE[SP]$ ; $SP := SP-1$ | $\binom{T}{T}$ | $(b)$ |
| **leq** $T$ | $STORE[SP-1] := STORE[SP-1] \leq_T STORE[SP]$ ; $SP := SP-1$ | $\binom{T}{T}$ | $(b)$ |
| **les** $T$ | $STORE[SP-1] := STORE[SP-1] <_T STORE[SP]$ ; $SP := SP-1$ | $\binom{T}{T}$ | $(b)$ |
| **grt** $T$ | $STORE[SP-1] := STORE[SP-1] >_T STORE[SP]$ ; $SP := SP-1$ | $\binom{T}{T}$ | $(b)$ |
| **neq** $T$ | $STORE[SP-1] := STORE[SP-1] \neq_T STORE[SP]$ ; $SP := SP-1$ | $\binom{T}{T}$ | $(b)$ |

# P-instructions for load/store

| Instr. | Meaning | Cond. | Res. |
|---|---|---|---|
| **ldo** $T\,q$ | $SP := SP + 1;$<br>$STORE[SP] := STORE[\mathsf{q}]$ | $q \in [0, maxstr]$ | $(T)$ |
| **ldc** $T\,q$ | $SP := SP + 1;$<br>$STORE[SP] := q$ | $Typ(q) = T$ | $(T)$ |
| **ind** $T$ | $STORE[SP] := STORE[STORE[SP]]$ | $(a)$ | $(T)$ |
| **sro** $T\,q$ | $STORE[q] := STORE[SP];$<br>$SP := SP - 1$ | $(T)$<br>$q \in [0, maxstr]$ | |
| **sto** $T$ | $STORE[STORE[SP-1]] := STORE[SP];$<br>$SP := SP - 2$ | $\begin{pmatrix} a \\ T \end{pmatrix}$ | |



# Code Generation

- Assumptions about the input program:
  - No errors (syntax, types)
  - Structure and type information is available
  - No procedures (for now)

- $\rho(v)$ is the relative address of program variable $v$

- Invariant $I_0$ about the state of the P-machine:
  Let $code_R\ e\ \rho = is$,
  let $sp$ be the value of $SP$ before the execution of $is$.
  The execution of $is$ will leave the value of $e$ in $STORE[sp+1]$, and $SP$'s value will be $sp+1$. $STORE$ is otherwise unchanged.

# The translation of assignments and expressions

| Function | | Condition |
|---|---|---|
| $code_R(e_1 = e_2)\ \rho$ | $= code_R\ e_1\ \rho; code_R\ e_2\ \rho;$ **equ** $T$ | $Typ(e_1) = Typ(e_2) = T$ |
| $code_R(e_1 \neq e_2)\ \rho$ | $= code_R\ e_1\ \rho; code_R\ e_2\ \rho;$ **neq** $T$ | $Typ(e_1) = Typ(e_2) = T$ |
| $\vdots$ | | |
| $code_R(e_1 + e_2)\ \rho$ | $= code_R\ e_1\ \rho; code_R\ e_2\ \rho;$ **add** $N$ | $Typ(e_1) = Typ(e_2) = N$ |
| $code_R(e_1 - e_2)\ \rho$ | $= code_R\ e_1\ \rho; code_R\ e_2\ \rho;$ **sub** $N$ | $Typ(e_1) = Typ(e_2) = N$ |
| $code_R(e_1 * e_2)\ \rho$ | $= code_R\ e_1\ \rho; code_R\ e_2\ \rho;$ **mul** $N$ | $Typ(e_1) = Typ(e_2) = N$ |
| $code_R(e_1 / e_2)\ \rho$ | $= code_R\ e_1\ \rho; code_R\ e_2\ \rho;$ **div** $N$ | $Typ(e_1) = Typ(e_2) = N$ |
| $code_R(-e)\ \rho$ | $= code_R\ e\ \rho;$ **neg** $N$ | $Typ(e) = N$ |
| $code_R\ x\ \rho$ | $= code_L\ x\ \rho;$ **ind** $T$ | $x$ is a variable of type $T$ |
| $code_R\ c\ \rho$ | $=$ **ldc** $T$ c | $c$ is a constant of type $T$ |
| $code(x := e)\ \rho$ | $= code_L\ x\ \rho;\ code_R\ e\ \rho;$ **sto** $T$ | $Typ(e) = T$, |
| | | $x$ is a variable |
| $code_L\ x\ \rho$ | $=$ **ldc** a $\rho(x)$ | $x$ is a variable |

# Correctness of the code generation scheme

Proof by induction of the invariant $I_0$

**Base cases:**

$code_R\ x\ \rho$ and $code_R\ c\ \rho$

**Inductive step:**

Example: $code_R(e_1{+}e_2)\,\rho = code_R\,e_1\,\rho; code_R\,e_2\,\rho;$ **add** $=$ $is$

Let $sp$ be the value of $SP$ before the execution of $is$

Ind. Assumptions:

- The execution of $code_R\ e_1\ \rho$ leaves the value of $e_1$ in $STORE[SP+1]$ and $SP$ has value $sp+1$.
- The execution of $code_R\ e_2\ \rho$ leaves the value of $e_2$ in $STORE[SP+2]$ and $SP$ has value $sp+2$.

Step:

- **add** adds the topmost values and leaves the result, i.e. the value of $e$ in $STORE[SP+1]$ and $SP$ has value $sp+1$.

# Example

Assume that $Typ(x) = int$ and $\rho(x) = 5$

$$
\begin{aligned}
&code(x := 3) \;\; \rho \\
&\quad = code_L \; x \; \rho; \; code_R \; 3 \; \rho; \; \textbf{sto} \; \mathsf{i} \\
&\quad = \textbf{ldc} \; \mathsf{a} \; 5; \; code_R(3) \; \rho; \; \textbf{sto} \; \mathsf{i} \\
&\quad = \textbf{ldc} \; \mathsf{a} \; 5; \; \textbf{ldc} \; \mathsf{i} \; 3; \; \textbf{sto} \; \mathsf{i}
\end{aligned}
$$

# Example 2.1

Assume that $\rho(a) = 5$, $\rho(b) = 6$, and $\rho(c) = 7$ and that

$$Typ(a) = Typ(b) = Typ(c) = Typ(b{*}c) = Typ(b{+}b{*}c) = int$$

$code(a := (b + (b * c)))\ \ \rho$
$=$ $code_L\ a\ \rho$; $code_R\ (b + (b * c))\ \rho$; **sto** i
$=$ **ldc** a 5; $code_R(b + (b * c))\ \rho$; **sto** i
$=$ **ldc** a 5; $code_R(b)\ \rho$; $code_R(b * c)\ \rho$; **add** i; **sto** i
$=$ **ldc** a 5; $code_L(b)\ \rho$; **ind** i; $code_R(b * c)\ \rho$; **add** i; **sto** i
$=$ **ldc** a 5; **ldc** a 6; **ind** i; $code_R(b * c)\ \rho$; **add** i; **sto** i
$=$ **ldc** a 5; **ldc** a 6; **ind** i; $code_R(b)\ \rho$; $code_R(c)\ \rho$; **mul** i; **add** i; **sto** i
$=$ **ldc** a 5; **ldc** a 6; **ind** i; $code_L(b)\ \rho$; **ind** i; $code_R(c)\ \rho$; **mul** i; **add** i; **sto** i
$=$ **ldc** a 5; **ldc** a 6; **ind** i; **ldc** a 6; **ind** i; $code_R(c)\ \rho$; **mul** i; **add** i; **sto** i
$=$ **ldc** a 5; **ldc** a 6; **ind** i; **ldc** a 6; **ind** i; $code_L(c)\ \rho$; **ind** i; **mul** i; **add** i; **sto** i
$=$ **ldc** a 5; **ldc** a 6; **ind** i; **ldc** a 6; **ind** i; **ldc** a 7; **ind** i; **mul** i; **add** i; **sto** i

# Interpretation of the generated code

q

```
ssp 9
sep 4
ldc a 5
ldc a 6
ind i
ldc a 6
ind i
ldc a 7
ind i
mul i
add i
sto i
```

# (Non-)Optimality of the generated code

- Example 1: $a := b$
  - Generated code:
    **ldc** a $\rho(a)$; **ldc** a $\rho(b)$; **ind** i; **sto**
  - Minimal code
    **ldo** i $\rho(b)$; **sro** i $\rho(a)$
- Example 2: $a := a * a;$
  - Generated code:
    **ldc** a $\rho(a)$
    **ldc** a $\rho(a)$
    **ind** i
    **ldc** a $\rho(a)$
    **ind** i
    **mul** i
    **sto**
  - Minimal code
    **ldo** i $\rho(a)$
    **dpl** i
    **mul** i
    **sro** i $\rho(a)$

# Summary

- An inductive definition of the generated code
- Assuming that the relative addresses of variables are known at compile-time
- The stack machine simplifies the task of handling complex expressions