Estimating the Performance of Cache Replacement Policies

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MeMoCode, June 2008

Outline

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Locality & Caches

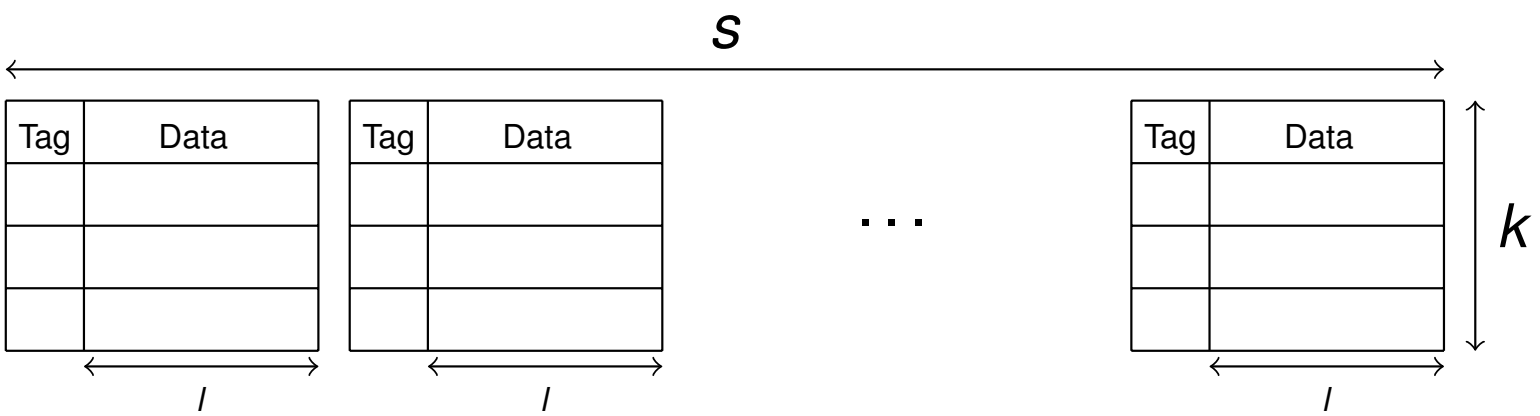
- Typical programs do not behave randomly, they exhibit
  - temporal locality
  - spatial locality

  in their memory access behavior

- Caches try to exploit locality
- Have great influence on system performance

Cache Parameters

Line size l	Size of blocks stored in cache
Associativity k	Number of lines one block may be stored in
Number of sets s	Each set comprises k lines
Capacity	Determined by $s \cdot k \cdot l$
Replacement policy	*next slide*



Replacement Policies

- The replacement policy determines the cache line whose contents are replaced upon a cache miss
- Examples:

LRU	Least recently used
FIFO	First-in first-out
PLRU	Approximation on LRU
MRU	Most recently used
...	

Problem Statement

- Cache performance depends on (locality of) executed program
- There are lots of design possibilities

Problem

Given a program, determine the best cache configuration.

Stack Histograms (Mattson et al. 1970)

- Concisely capture locality in memory access behavior
- Age of cacheable element e:
  number of accesses on different elements since last access on e
- Stack distance of a memory access:
  age of the accessed element just before the access
- Stack histogram:
  frequency distribution of stack distances

Example

Reference string	a	b	a	c	b	b	c	a
Stack distance	∞	∞	1	∞	2	0	1	2

Stack distance d	0	1	2	3	≥ 4
Rel. frequency f_d	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{2}{8}$	0	$\frac{3}{8}$

Estimation for LRU Is Easy

Mattson et al. (1970)

- LRU always retains the k most recently used elements
- Exactly the ages $0, \ldots, k-1$ are cached

Example

Associativity $k = 8$

d	0	1	2	3	4	5	6	7	8	9	10	11	12	$\geq$
f_d	$\longleftarrow$							$\longrightarrow$	$\longleftarrow$					$\longrightarrow$
	always cached								never cached					

$\Rightarrow$ Miss ratio $= 1 - \sum_{i=0}^{7} f_i$

For Other Policies It Is Not

Problem:

- In general, the ages of cached elements vary during execution
- Number and shape of combinations of ages depend on policy

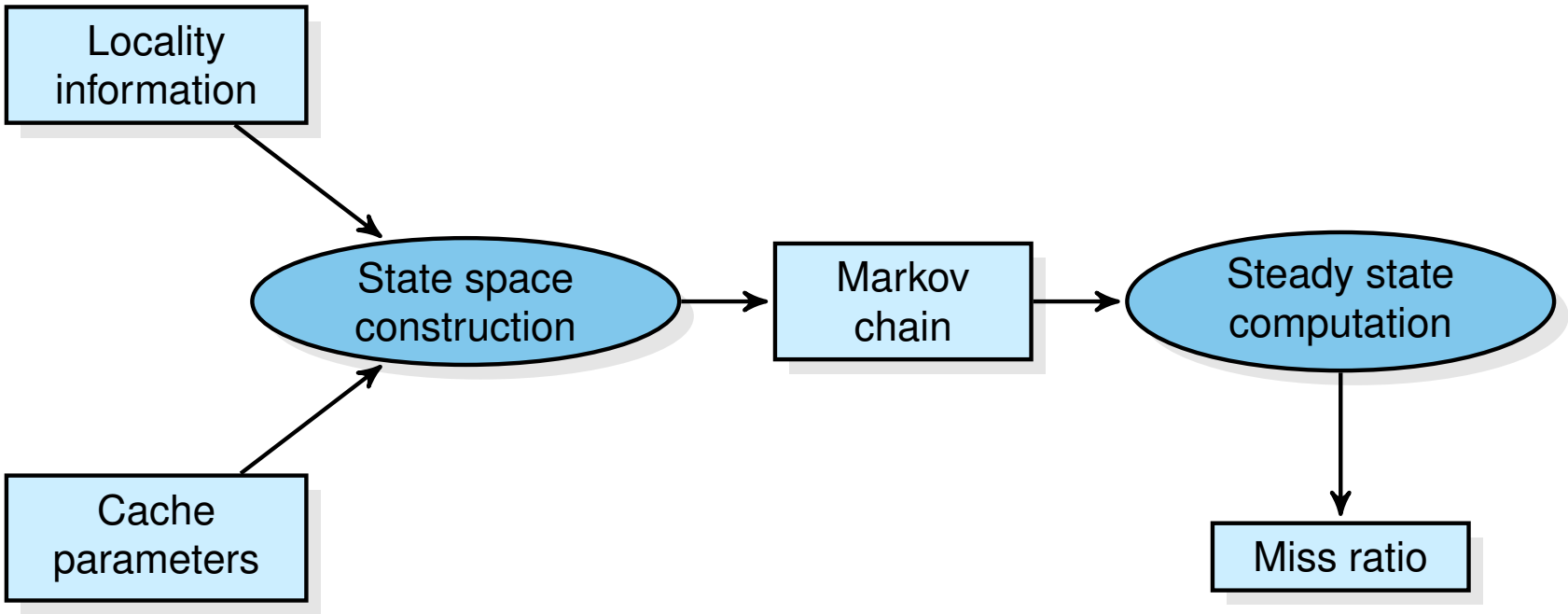
Idea: determine

- all possible combinations (state space)
- probability of each state
- probability of a cache miss in each state

Outline

Our Approach

- Static method
- Given . . .
  - a concise characterization of the program's locality
  - parameters of the cache architecture
- . . . we construct a Markov chain to estimate the miss ratio



Contributions

- Replacement policy is taken into account
- Policy tables
  - uniformly represent replacement policies
  - ease automatic model construction
- History stack histograms
  - extend well-known stack histograms
  - increase precision of estimates

Policy Tables

- Uniform representation for a class of replacement policies
- Class contains LRU, PLRU, FIFO, MRU
- Eases automatic model construction

π_0	0	1	2	3	4	5	6	7
π_1	0	1	2	3	4	5	6	7
π_2	0	1	2	3	4	5	6	7
π_3	0	1	2	3	4	5	6	7
π_4	0	1	2	3	4	5	6	7
π_5	0	1	2	3	4	5	6	7
π_6	0	1	2	3	4	5	6	7
π_7	0	1	2	3	4	5	6	7
π_m	1	2	3	4	5	6	7	0

8-way FIFO

π_0	1	2	3	4	5	6	7	0
π_1	0	2	3	4	5	6	7	1
π_2	0	1	3	4	5	6	7	2
π_3	0	1	2	4	5	6	7	3
π_4	0	1	2	3	5	6	7	4
π_5	0	1	2	3	4	6	7	5
π_6	0	1	2	3	4	5	7	6
π_7	0	1	2	3	4	5	6	7
π_m	1	2	3	4	5	6	7	0

8-way LRU

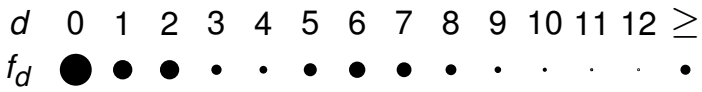
- *Policy state* is virtual order on cache lines

  $[3, 2, 1, 0, 4, 5, 6, 7]$
- *Updates* are specified by permutations
  - π_i for cache-hit on position i
  - π_m for cache-miss

$$[3, 2, 1, 0, 4, 5, 6, 7] \xrightarrow{\pi_4} [3, 2, 1, 0, 5, 6, 7, 4]$$

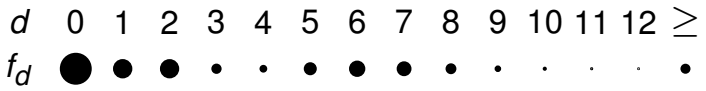
History Stack Histograms

Circle plots of stack histograms

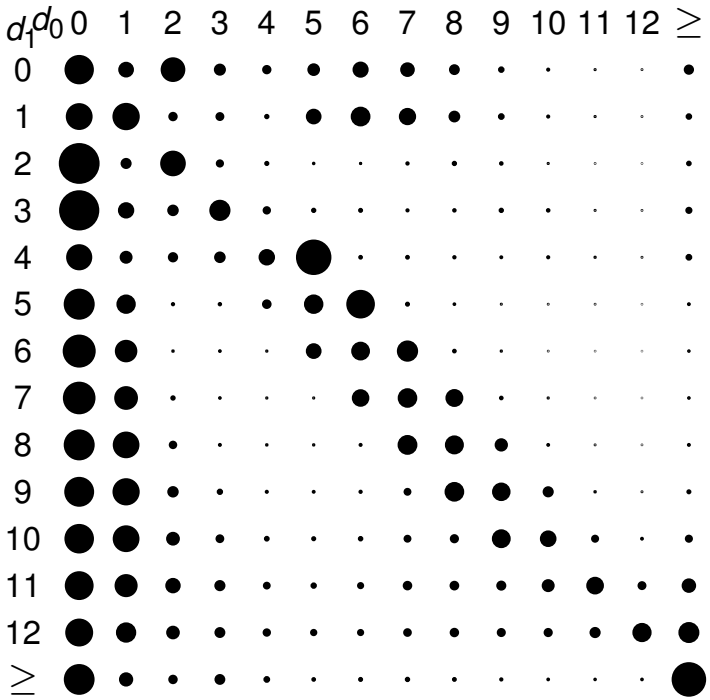


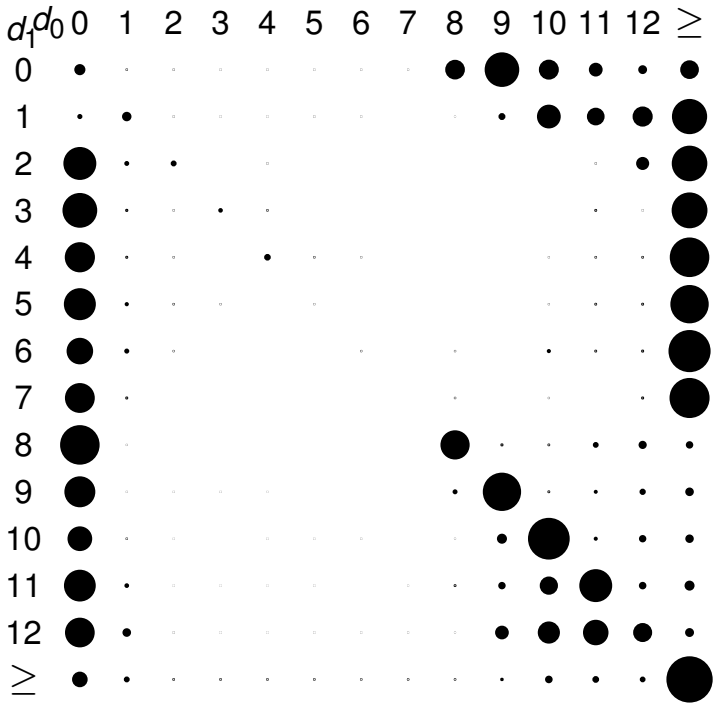
History Stack Histograms

Circle plots of stack histograms



History stack histograms consider context of occurring stack distances





$\Rightarrow$ increases precision of estimates

Model Construction: States (1)

- Concrete cache set state: tuple of elements and status bits
- Two FIFO states: $[a, \underline{b}, c, d]$ and $[d, a, \underline{b}, c]$

- Only relative order w.r.t. replacment matters
- Abstract from physical positions

$\Rightarrow$ Normalized tuple: $[b, c, d, a]$

- Only ages of elements matter
- Abstract from names

$\Rightarrow$ Normalized tuple of ages, e.g. $[4, 1, 5, 0]$

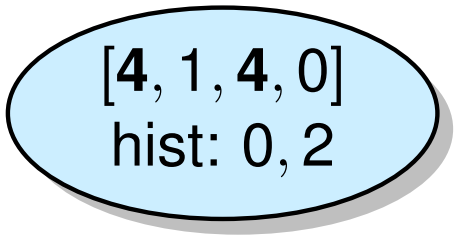
Model Construction: States (2)

- Problem: arbitrarily high ages $\Rightarrow$ infinite model
- Introduce cutoff-age, abstract from high ages

$\Rightarrow$ $\alpha([4,1,5,0],[6,1,4,0]) = [\mathbf{4},1,\mathbf{4},0]$

- Disambiguate states by history to fit history stack histograms

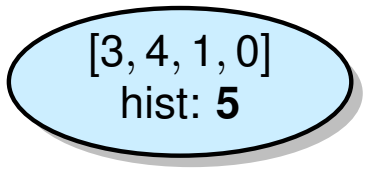
$\Rightarrow$ Abstract cache set state:



Model Construction: Transitions

State space construction:

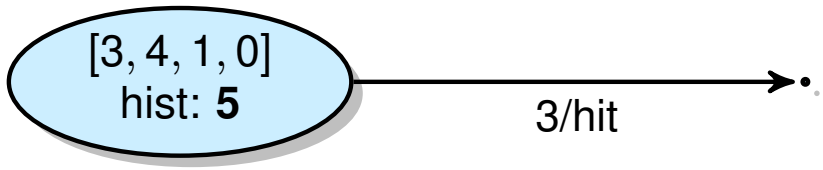
- Pick unexplored abstract state s



Model Construction: Transitions

State space construction:

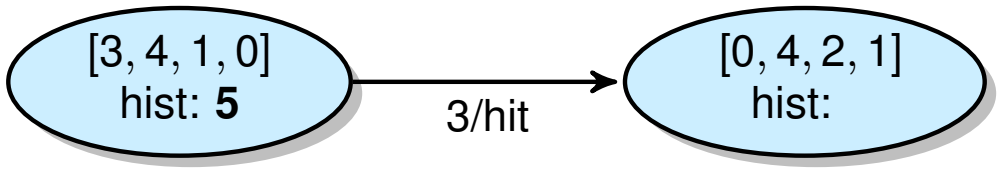
- Pick unexplored abstract state s
- Enumerate all possible accesses (on ages)



Model Construction: Transitions

State space construction:

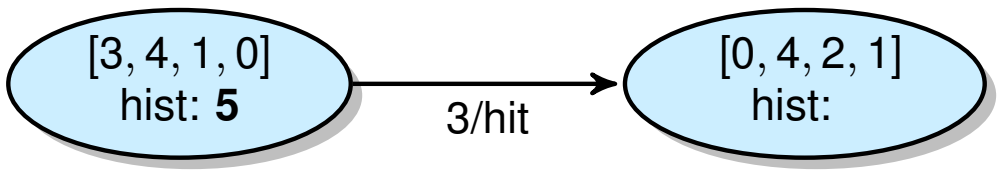
- Pick unexplored abstract state s
- Enumerate all possible accesses (on ages)
- For each successor state s'
  - Update ages



Model Construction: Transitions

State space construction:

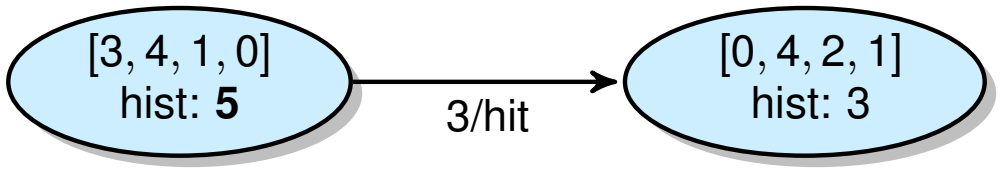
- Pick unexplored abstract state s
- Enumerate all possible accesses (on ages)
- For each successor state s'
  - Update ages
  - Update order (apply permutation from policy table)



Model Construction: Transitions

State space construction:

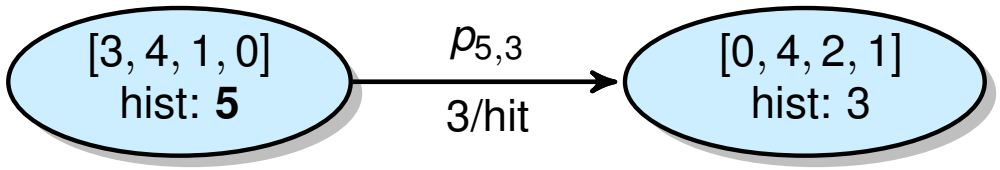
- Pick unexplored abstract state s
- Enumerate all possible accesses (on ages)
- For each successor state s'
  - Update ages
  - Update order (apply permutation from policy table)
  - Adjust history



Model Construction: Transitions

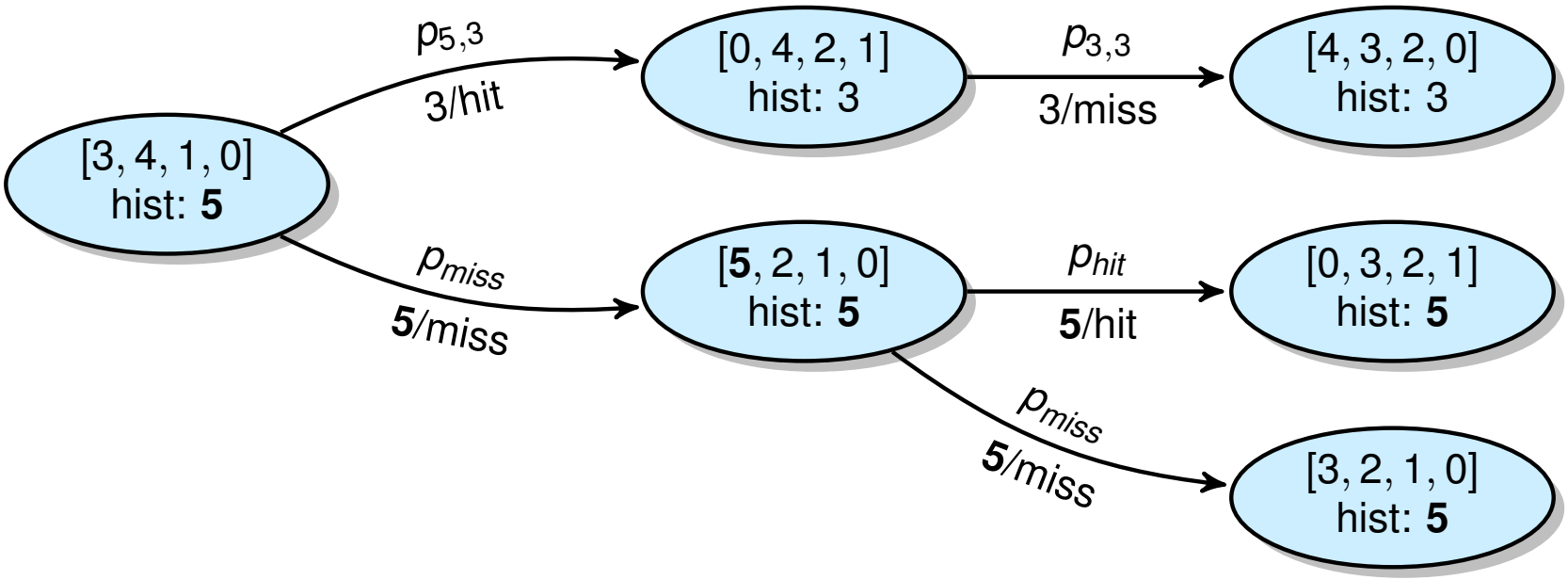
State space construction:

- Pick unexplored abstract state s
- Enumerate all possible accesses (on ages)
- For each successor state s'
  - Update ages
  - Update order (apply permutation from policy table)
  - Adjust history
  - Compute probability for transition $s \rightarrow s'$ from history stack histogram



Model Construction: Example

4-way FIFO, history length 1, cutoff age 5



Computing the Miss-Ratio Estimate

1. Compute steady state probabilities $p(s_i)$ of the Markov chain
   - e.g. with the Gauss-Seidel algorithm
2. Compute miss probabilities $p(m_i)$
   - simple sum of values in history stack histogram

$\Rightarrow$ Miss ratio $= \sum_i p(s_i) \cdot p(m_i)$

Outline

Evaluation: Precision

- Shared 2nd level caches with different parameter combinations
- C programs of SPEC CPU2000
- Evaluated against full simulation runs (SimpleScalar)

- Average absolute error between 0.18% and 2.92%
- Better to spend resources on history than on higher cutoff age

- Example: 8-way associative, 1024 sets, 32 bytes line size

	LRU	PLRU	FIFO	MRU	RAND
Average error	0	0.23	0.58	2.92	1.61

Evaluation: Runtime

- k=4 is handled easily
- k=8 often requires cheaper model parameters
  i.e. stronger abstraction / less precision

- Example: Most expensive model parameters
  for $k = 8$ and 2 GB main memory

	PLRU	FIFO	MRU	RAND
#states	4,166,048	8,216,745	4,129,362	2,687,856
∅runtime [s]	386	440	263	234

- Comparison: ∅simulation time was 18 hours

Limitations / Future Work

- Sometimes precision is limited by resources/model parameters, e.g. 8-FIFO or 8-RAND with history
- History length is currently limited to 1.
  ≥ 2 requires $>$ 2GB main memory or stronger abstractions
- Implementation limited to policies in policy table class.
  Others (e.g. 2Q, EE-LRU, ARC) conceivable
  but most likely require different abstractions

Related Work

Mattson, Gecsei, Slutz, and Traiger
Evaluation techniques for storage hierarchies
IBM Systems Journal, 9(2), 1970.

Guo and Solihin
An analytical model for cache replacement policy performance
SIGMETRICS Perf. Eval. Rev., 34(1), 2006.

- Mattson et al. limited to LRU, but seminal work and still useful
- Guo and Solihin
  - policies directly specified in terms of probability functions
  - hence method more efficient
  - but real-world policies (PLRU, FIFO, MRU) impossible

Outline

Conclusions

- Static method for miss-ratio estimation
  - stochastic model yields precise estimates
  - allows for considering real-world policies
- History stack histograms
  - extend well-known stack histograms
  - increase precision of estimates
- Policy tables
  - provide uniform representation of policies