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Development of Safety-Critical Embedded Systems

Static Program Analysis

Winter Semester 2012/2013

Slides based on:

- H. Seidl, R. Wilhelm, S. Hack: Compiler Design, Volume 3, Analysis and Transformation, Springer Verlag, 2012
- F. Nielson, H. Riis Nielson, C. Hankin: Principles of Program Analysis, Springer Verlag, 1999
- R. Wilhelm, B. Wachter: Abstract Interpretation with Applications to Timing Validation. CAV 2008: 22-36
- Helmut Seidl's slides

A Short History of Static Program Analysis

- Early high-level programming languages were implemented on very small and very slow machines.
- Compilers needed to generate executables that were extremely efficient in space and time.
- Compiler writers invented efficiency-increasing program transformations, wrongly called optimizing transformations.
- Transformations must not change the semantics of programs.
- Enabling conditions guaranteed semantics preservation.
- Enabling conditions were checked by static analysis of programs.

Theoretical Foundations of Static Program Analysis

- Theoretical foundations for the solution of recursive equations: Kleene (30s), Tarski (1955)
- Gary Kildall (1972) clarified the lattice-theoretic foundation of data-flow analysis.
- Patrick Cousot (1974) established the relation to the programming-language semantics.

Static Program Analysis as a Verification Method

- Automatic method to derive invariants about program behavior, answers questions about program behavior:
  - will index always be within bounds at program point p?
  - will memory access at p always hit the cache?
- answers of sound static analysis are correct, but approximate: don't know is a valid answer!
- analyses proved correct wrt. language semantics,

Proposed Lectures Content:

1. Introductory example: rules-of-sign analysis
2. theoretical foundations: lattices
3. an operational semantics of the language
4. another example: constant propagation
5. relating the semantics to the analysis—correctness proofs
6. Further static analyses in compilers: Elimination of superfluous computations
   - $\rightarrow$ available expressions
   - $\rightarrow$ live variables
   - $\rightarrow$ array-bounds checks
7. timing (WCET) analysis
8. analysis for runtime errors

1 Introduction

... in this course and in the Seidl/Wilhelm/Hack book:

a simple imperative programming language with:

- variables // registers
- $R = e;$ // assignments
- $R = M[e];$ // loads
- $M[e_1] = e_2;$ // stores
- if (e) s_1 else s_2 // conditional branching
- goto $L;$ // no loops

An intermediate language into which (almost) everything can be translated.

In particular, no procedures. So, only intra-procedural analyses!

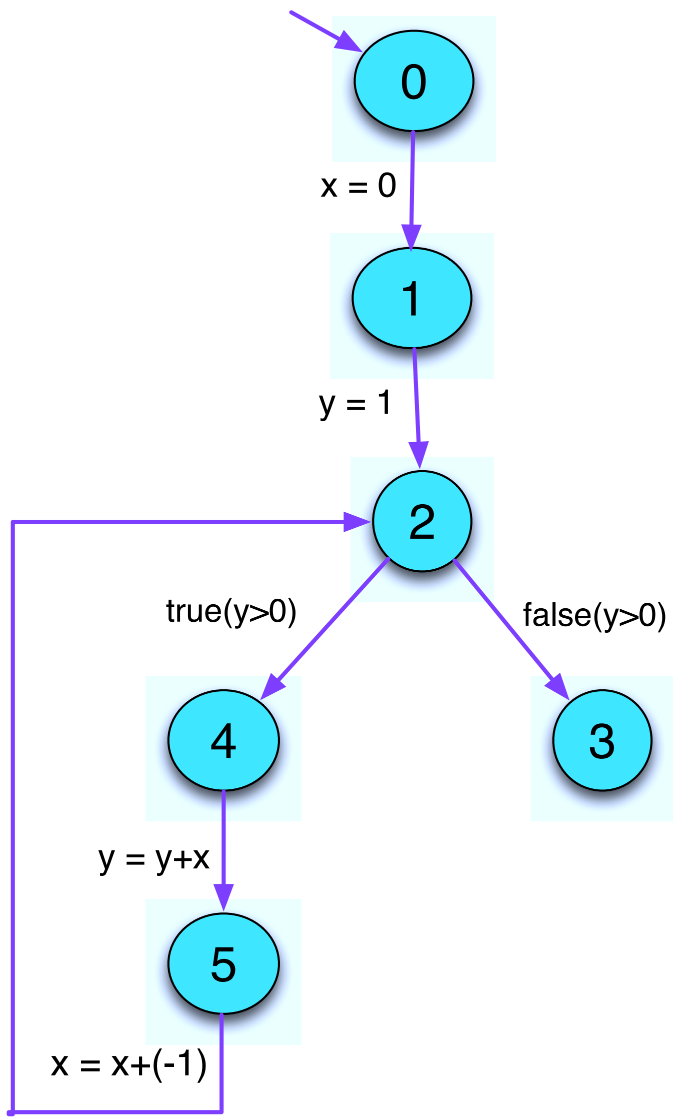
2 Example — Rules-of-Sign Analysis

Problem: Determine at each program point the sign of the values of all variables of numeric type.

Example program:

```
x = 0;
y = 1;
while (y > 0) do
    y = y + x;
    x = x + (-1);
```

Program representation as *control-flow graphs*



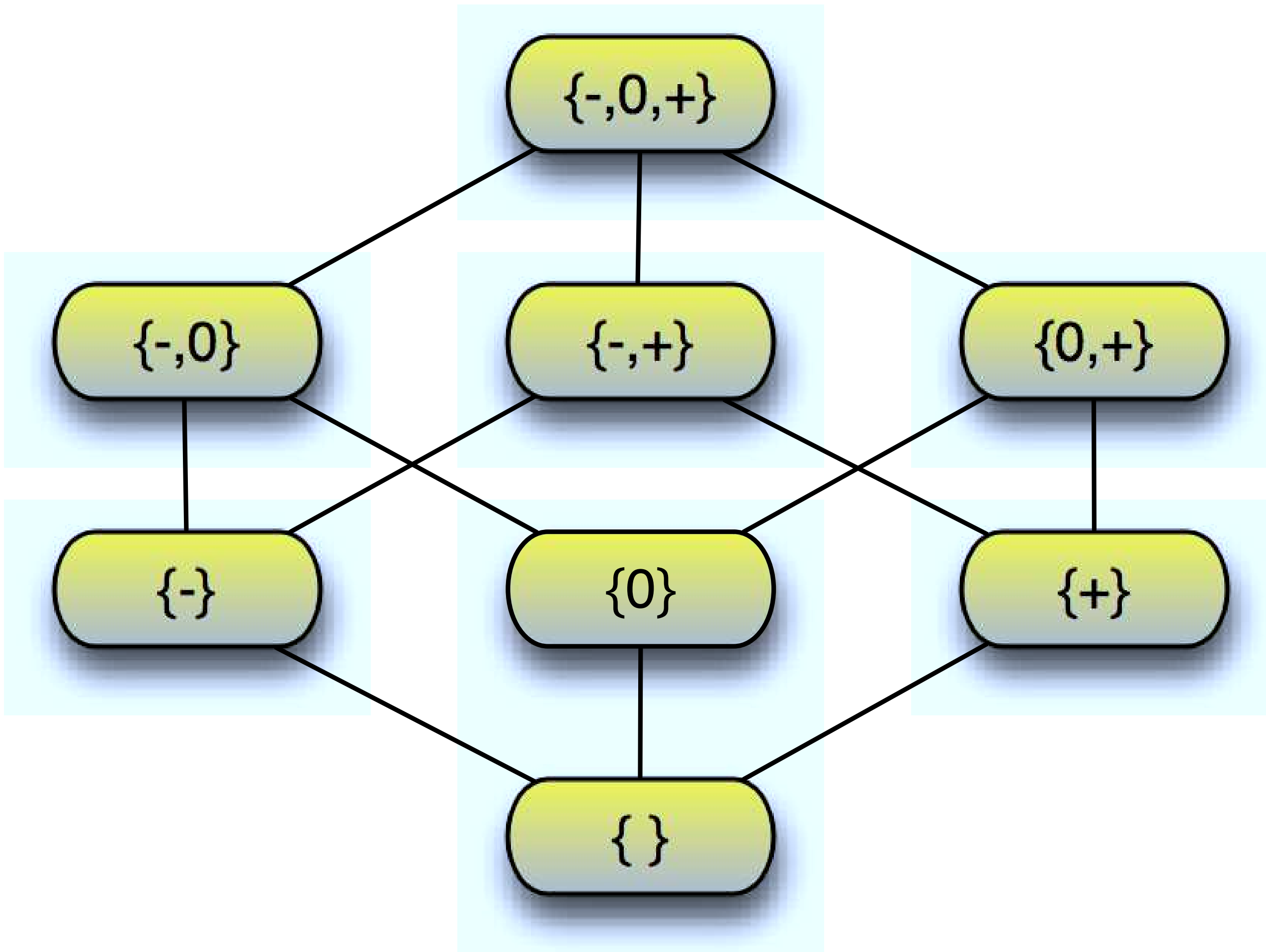
What are the ingredients that we need?

More ingredients?

All the ingredients:

- a set of information elements, each a set of possible signs,
- a partial order, "$\sqsubseteq$", on these elements, specifying the "relative strength" of two information elements,
- these together form the abstract domain, a lattice,
- functions describing how signs of variables change by the execution of a statement, abstract edge effects,
- these need an abstract arithmetic, an arithmetic on signs.

We construct the abstract domain for single variables starting with the lattice $Signs = 2^{\{-,0,+\}}$ with the relation "$\sqsubseteq$" ="$\subseteq$".



The analysis should ”bind” program variables to elements in $Signs$.

So, the abstract domain is $\mathbb{D} = (\mathit{Vars} \to \mathit{Signs})_\perp$, a Sign-environment.

$\perp \in \mathbb{D}$ is the function mapping all arguments to $\{\}$.

The partial order on $\mathbb{D}$ is $\qquad D_1 \sqsubseteq D_2 \quad$ iff

$D_1 = \perp \qquad\qquad$ or

$D_1\, x \subseteq D_2\, x \qquad (x \in \mathit{Vars})$

Intuition?

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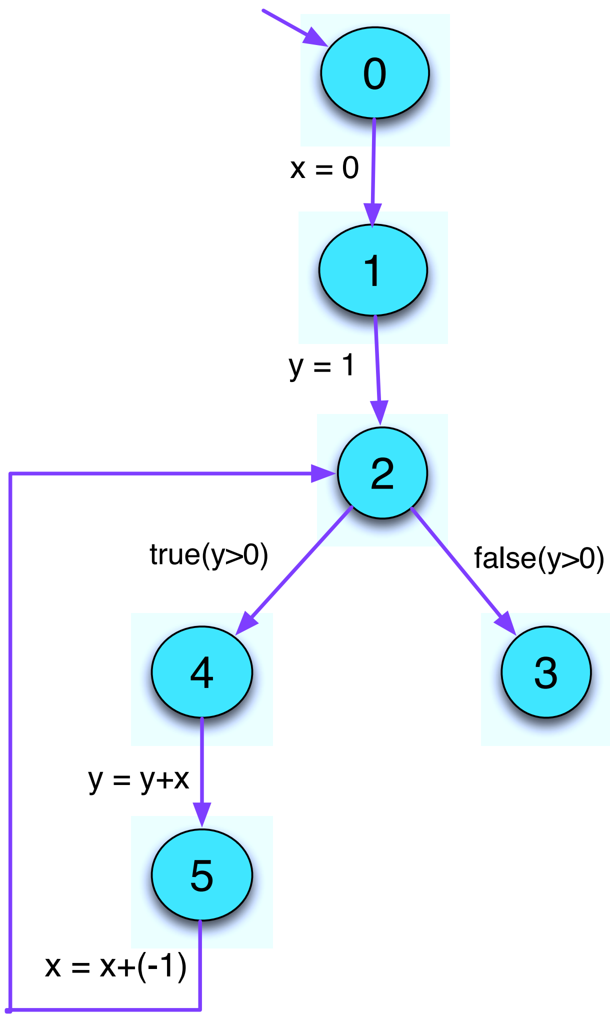
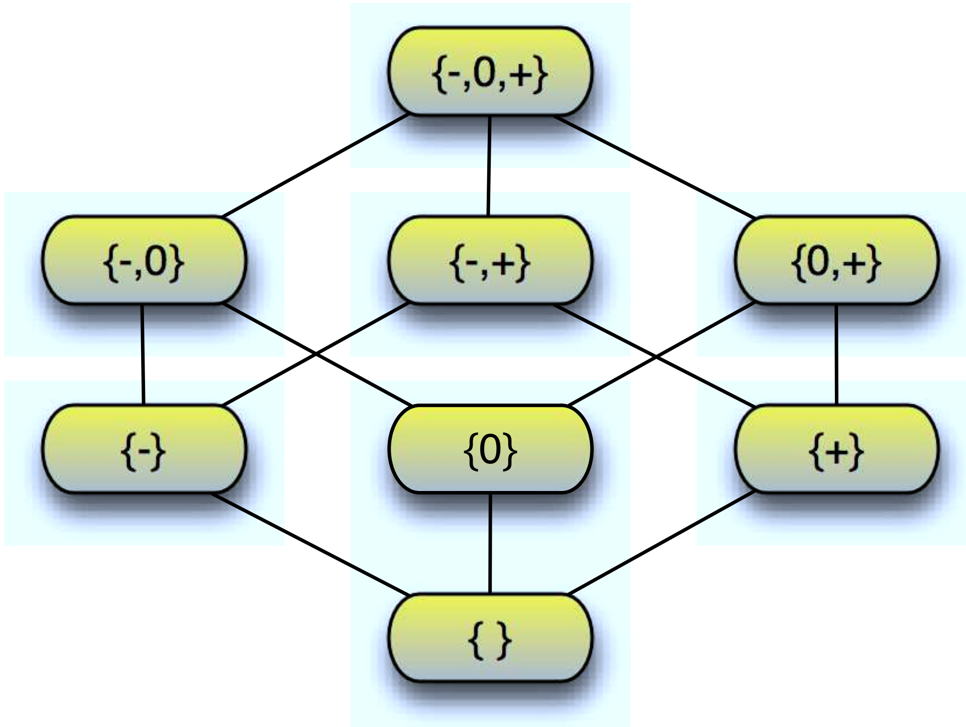
Intuition?

D_1 is at least as precise as D_2 since D_2 admits at least as many signs as D_1

How did we analyze the program?

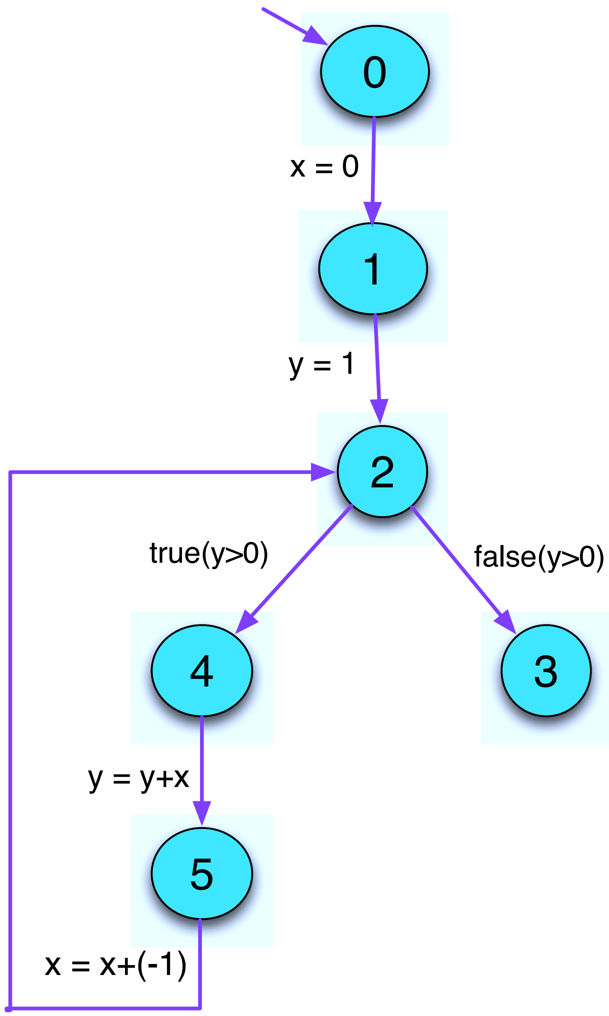


In particular, how did we walk the lattice for y at program point 5?



How is a solution found?

Iterating until a fixed-point is reached



0		1		2		3		4		5	
x	y	x	y	x	y	x	y	x	y	x	y

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- For some sub-expressions, the analysis may yield $\{+, -, 0\}$, which means, it couldn't find out.
- We replace the concrete operators $\square$ working on values by abstract operators $\square^\sharp$ working on signs:
- The abstract operators allow to define an abstract evaluation of expressions:

$$[\![e]\!]^\sharp \;:\; (Vars \to Signs) \to Signs$$

Determining the sign of expressions in a Sign-environment works as follows:

$$
\begin{aligned}
[\![c]\!]^\sharp\, D &= \begin{cases} \{+\} & \text{if c > 0} \\ \{-\} & \text{if c < 0} \\ \{0\} & \text{if c = 0} \end{cases} \\
[\![v]\!]^\sharp &= D(v) \\
[\![e_1 \,\square\, e_2]\!]^\sharp\, D &= [\![e_1]\!]^\sharp\, D \,\square^\sharp\, [\![e_2]\!]^\sharp\, D \\
[\![\square e]\!]^\sharp\, D &= \square^\sharp [\![e]\!]^\sharp\, D
\end{aligned}
$$

Abstract operators working on signs (Addition)

$+^{\#}$	{0}	{+}	{-}	{-, 0}	{-, +}	{0, +}	{-, 0, +}
{0}	{0}	{+}					
{+}							
{-}							
{-, 0}							
{-, +}							
{0, +}							
{-, 0, +}	{-, 0, +}						

Abstract operators working on signs (Multiplication)

$\times^{\#}$	{0}	{+}	{-}	{-, 0}	{-, +}	{0, +}	{-, 0, +}
{0}	{0}	{0}					
{+}							
{-}							
{-, 0}							
{-, +}							
{0, +}							
{-, 0, +}	{0}						

Abstract operators working on signs (unary minus)

$-^{\#}$	{0}	{+}	{-}	{-, 0}	{-, +}	{0, +}	{-, 0, +}
	{0}	{-}	{+}	{+, 0}	{-, +}	{0, -}	{-, 0, +}

Working an example:

$$D = \{x \mapsto \{+\}, y \mapsto \{+\}\}$$

$$\begin{aligned}
[\![x + 7]\!]^\sharp\, D &= [\![x]\!]^\sharp\, D \;+^\sharp\; [\![7]\!]^\sharp\, D \\
&= \{+\} \;+^\sharp\; \{+\} \\
&= \{+\} \\
[\![x + (-y)]\!]^\sharp\, D &= \{+\} \;+^\sharp\; (-^\sharp [\![y]\!]^\sharp\, D\,) \\
&= \{+\} \;+^\sharp\; (-^\sharp \{+\}) \\
&= \{+\} \;+^\sharp\; \{-\} \\
&= \{+, -, 0\}
\end{aligned}$$

Thus, we obtain the following effects of edges $[\![lab]\!]^\sharp$:

$$\begin{array}{lll}
[\![;]\!]^\sharp\, D & = & D \\
[\![\text{true}\,(e)]\!]^\sharp\, D & = & D \\
[\![\text{false}\,(e)]\!]^\sharp\, D & = & D \\
[\![x = e;]\!]^\sharp\, D & = & D \oplus \{x \mapsto [\![e]\!]^\sharp\, D\} \\
[\![x = M[e];]\!]^\sharp\, D & = & D \oplus \{x \mapsto \{+,-,0\}\} \\
[\![M[e_1] = e_2;]\!]^\sharp\, D & = & D
\end{array}$$

... whenever $D \neq \bot$