Lecture 14

Real-Time Scheduling

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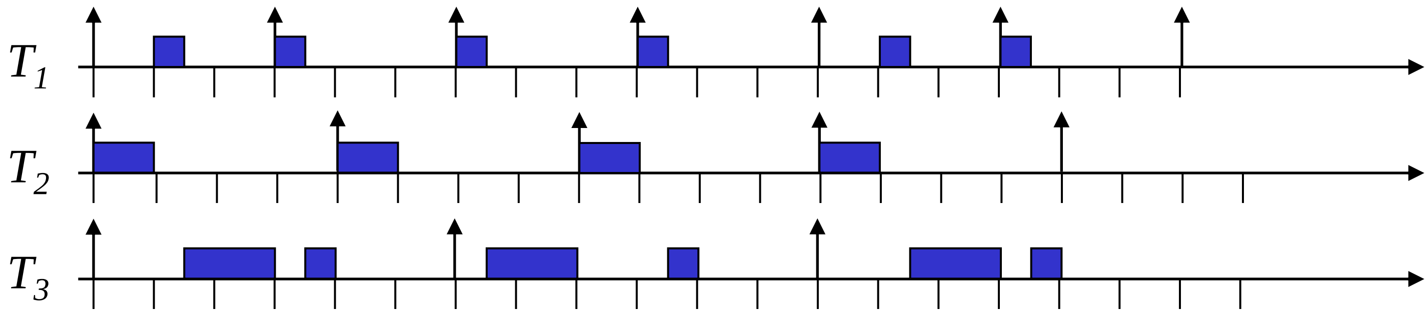
Deadline Monotonic Scheduling

- Let each process have a unique priority P_i based on its relative deadline d_i.
- We assume that the shorter the deadline, the higher the priority, ie $d_i < d_j \Leftrightarrow P_i > P_j$.
- Same as rate monotonic, if each task's relative deadline equals its period.
- Example schedule: T_1 with $\pi_1 = d_1 = 3$ and $c_1 = 0.5$, T_2 with $\pi_2 = 4$, $d_2 = 2$ and $c_2 = 1$ and T_3 with $\pi_3 = d_3 = 6$ and $c_3 = 2$.



Schedulability Analysis

- The rate monotonic schedulability test can be applied also to deadline monotonic scheduling, by reducing periods to relative deadlines:

$$\sum_{i=1}^{N} \frac{c_i}{d_i} \leq N(2^{\frac{1}{N}} - 1)$$

- However, this test significantly overestimates the workload on the processor.
- Observations:
  - The worst-case processor demand occurs when all tasks are released at their critical instants.
  - For each task T_i the sum of its processing time and the interference (preemption) imposed by higher priority tasks must be less than or equal to its deadline d_i.

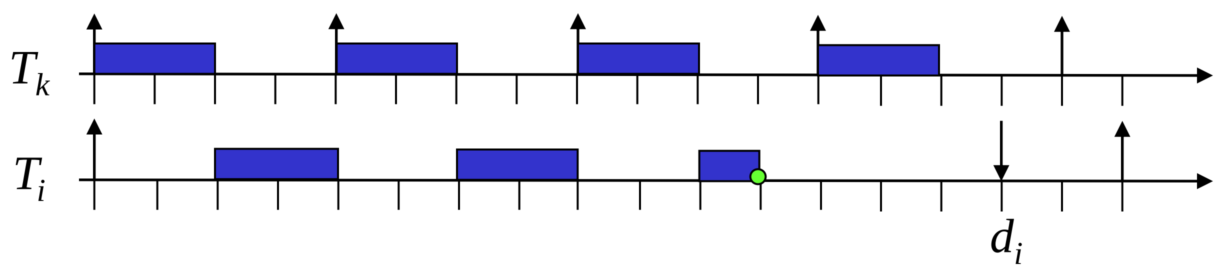
Schedulability Analysis

- Assume that tasks are ordered by increasing relative deadlines: $i<j \Leftrightarrow d_i<d_j \Leftrightarrow P_i>P_j$.
- Then a task set $\Gamma = \{ T_i \}_{i=1..N}$ is schedulable if the following condition is satisfied:

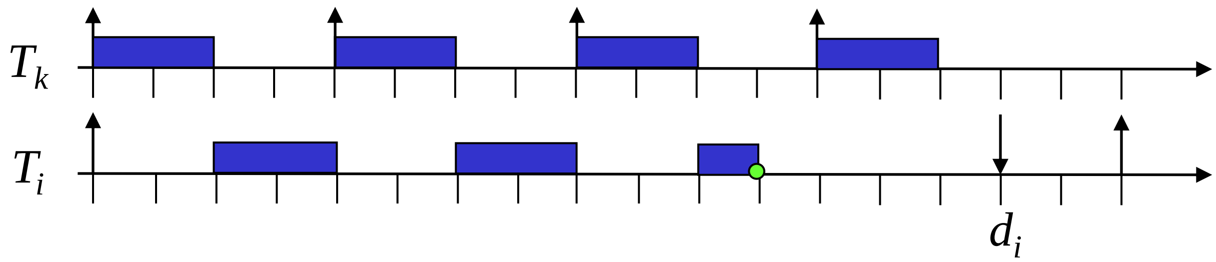
$$\forall 1 \le i \le N : \; c_i + I_i \le d_i$$

- where I_i is a measure of the interference of T_i, which can be computed as the sum of the processing times of all higher-priority tasks released before d_i:

$$I_i = \sum_{j=1}^{i-1} \left\lceil \frac{d_i}{\pi_j} \right\rceil c_j$$



Schedulability Analysis



- Note that this test is sufficient but not necessary.
- I_i is calculated by assuming that each higher-priority task exactly interferes $\left\lceil \frac{d_i}{\pi_j} \right\rceil$ times during the execution time of T_i. However, since T_i may terminate earlier, the actual interference may be smaller.
- A sufficient and necessary schedulability test for DM must take the exact interleaving of higher-priority tasks into account for each process.

Response Time Analysis

- The longest response time R_i of a periodic task T_i is computed as the sum of its computation time and the interference due to preemption by higher-priority tasks at the critical instant.

$$R_i = c_i + I_i$$

where

$$I_i = \sum_{j=1}^{i-1} \left\lceil \frac{R_i}{\pi_j} \right\rceil c_j$$

such that

$$R_i = c_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i}{\pi_j} \right\rceil c_j \qquad (*)$$

The worst-case response time is the smallest value of R_i that satisfies Eq.(*).

Response Time Analysis

- Solution: Fixed point iteration.
- Let $R_i^{(k)}$ be the k-th value of R_i and let $I_i^{(k)}$ be the interference on task T_i in the interval $[0, R_i^{(k)}]$:

$$I_i^{(k)} = \sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(k)}}{\pi_j} \right\rceil c_j$$

- Let $R_i^{(0)}$ be the first point in time that T_i could possibly complete:

$$R_i^{(0)} = \sum_{j=1}^{i} c_j$$

For $k > 0$ repeatedly compute $R_i^{(k+1)}$ until $R_i^{(k+1)} = R_i^{(k)}$.

$$R_i^{(k+1)} = I_i^{(k)} + c_i = \left(\sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(k)}}{\pi_j} \right\rceil c_j\right) + c_i$$

Response Time Analysis

- The task set is schedulable if $R_i \leq d_i$ holds for the fixed point R_i.
- RTA is necessary and sufficient.
- Let N be the number of tasks and m the number of iterations of the fixed point algorithm. Then the complexity of the RTA algorithm is $O(Nm)$.

Response Time Analysis – Example

- Consider the task set on the right side.
- Assume T1-T3 have been shown to be schedulable. Is also the task set with T4 schedulable?

Process	Period π	WCET c	Deadline d
T1	4	1	3
T2	5	1	4
T3	6	2	5
T4	11	1	10

$$R_4^{(0)} = \sum_{j=1}^{4} c_j = 1+1+2+1=5$$

$$R_4^{(1)} = (\sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(0)}}{\pi_j} \right\rceil c_j) + c_i = \left\lceil \frac{R_4^{(0)}}{\pi_1} \right\rceil c_1 + \left\lceil \frac{R_4^{(0)}}{\pi_2} \right\rceil c_2 + \left\lceil \frac{R_4^{(0)}}{\pi_3} \right\rceil c_3 + c_4 = \left\lceil \frac{5}{4} \right\rceil 1 + \left\lceil \frac{5}{5} \right\rceil 1 + \left\lceil \frac{5}{6} \right\rceil 2 + 1 = 2+1+2+1=6$$

$$R_4^{(2)} = (\sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(1)}}{\pi_j} \right\rceil c_j) + c_i = \left\lceil \frac{6}{4} \right\rceil 1 + \left\lceil \frac{6}{5} \right\rceil 1 + \left\lceil \frac{6}{6} \right\rceil 2 + 1 = 2+2+2+1=7$$

$$R_4^{(3)} = (\sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(2)}}{\pi_j} \right\rceil c_j) + c_i = \left\lceil \frac{7}{4} \right\rceil 1 + \left\lceil \frac{7}{5} \right\rceil 1 + \left\lceil \frac{7}{6} \right\rceil 2 + 1 = 2+2+4+1=9$$

$$R_4^{(4)} = (\sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(3)}}{\pi_j} \right\rceil c_j) + c_i = \left\lceil \frac{9}{4} \right\rceil 1 + \left\lceil \frac{9}{5} \right\rceil 1 + \left\lceil \frac{9}{6} \right\rceil 2 + 1 = 3+2+4+1=10$$

$$R_4^{(5)} = (\sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(4)}}{\pi_j} \right\rceil c_j) + c_i = \left\lceil \frac{10}{4} \right\rceil 1 + \left\lceil \frac{10}{5} \right\rceil 1 + \left\lceil \frac{10}{6} \right\rceil 2 + 1 = 3+2+4+1=10$$

Earliest Deadline First

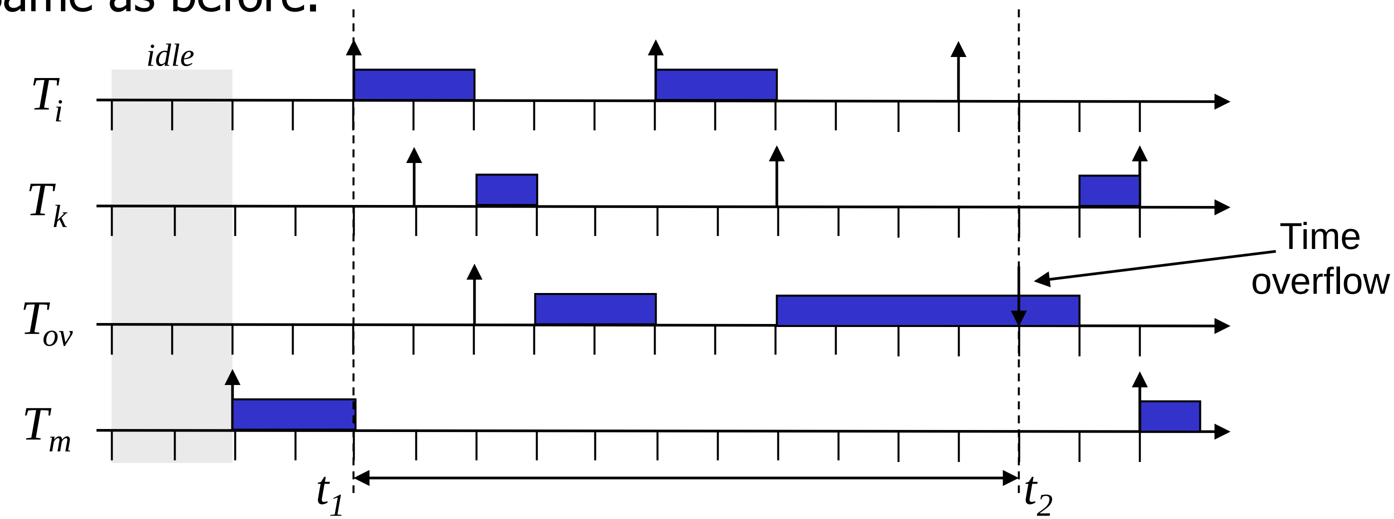
- Earliest Deadline First (EDF) is a dynamic scheduling scheme that selects tasks according to their absolute deadline. Tasks with earlier deadlines will be executed at higher priorities.
- So far: d_i relative deadline, ie the time between T_i becoming available and the time until which T_i has to finish execution.
- Let $T_{i,j}$ denote the j-th instance of task T_i.
- Let $r_{i,j}$ be the release time of the j-th instance of task T_i .
- Let Φ_i denote the phase of task T_i, ie the release time of its first instance ($\Phi_i = r_{i,1}$).
- $d_{i,j}$ denotes the absolute deadline of the j-th instance of task T_i which is given by $d_{i,j} = \Phi_i + (j - 1)\,\pi_i + d_i$
- EDF assumes tasks are preemptive; tasks can be periodic or aperiodic.

Earliest Deadline First

- Theorem: A set of periodic tasks is schedulable with EDF if and only if

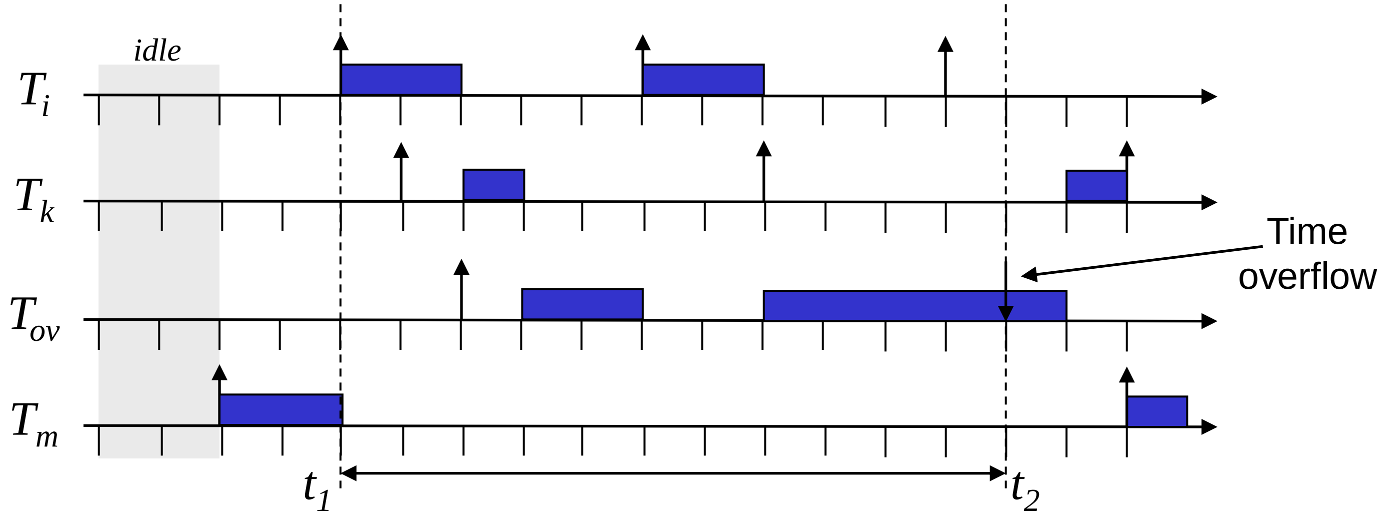
$$U = \sum_{i=1}^{N} \frac{c_i}{\pi_i} \leq 1$$

- Proof:
  - "⇒" Same as before.
  - "⇐"



  - Assume that $U \leq 1$ and the task set is not schedulable.
  - Let t_2 be the instant where the deadline violation occurs.
  - Let $[t_1, t_2]$ be the longest interval of continuous utilization before the overflow, such that only instances with deadline $\leq t_2$ are executed in $[t_1, t_2]$.

Earliest Deadline First



- Let $C_p(t_1,t_2)$ be the total computation time demanded by periodic tasks in $[t_1,t_2]$. Then

$$C_p(t_1,t_2) = \sum_{r_k \geq t_1, d_k \leq t_2} c_k = \sum_{i=1}^{N} \left\lfloor \frac{t_2 - t_1}{\pi_i} \right\rfloor c_i \leq \sum_{i=1}^{N} \frac{t_2 - t_1}{\pi_i} c_i = (t_2 - t_1)U$$

- Since a deadline is missed at t_2, $C_p(t_1,t_2)$ must be greater than the available processor time $t_2 - t_1$. Thus

$$t_2 - t_1 < C_p(t_1,t_2) \leq (t_2 - t_1)U \iff U > 1 \lightning$$

EDF vs. RM

- Fixed-priority scheduling is easier to implement since priorities are static.
- Dynamic schemes require a more complex run-time system which will have higher overhead.
- It is easier to incorporate processes without deadlines into RM; giving a process an arbitrary deadline is more artificial.

EDF vs. RM

- During overload situations
  - RM is more predictable. Low priority processes miss their deadlines first.
  - EDF is unpredictable; a domino effect can occur in which a large number of processes miss deadlines.
    - To counter this detrimental domino effect, many on-line schemes have two mechanisms:
      - an admissions control module that limits the number of processes that are allowed to compete for the processors, and
      - an EDF dispatching routine for those processes that are admitted
    - An ideal admissions algorithm prevents the processors getting overloaded so that the EDF routine works effectively

Resource Access Protocols

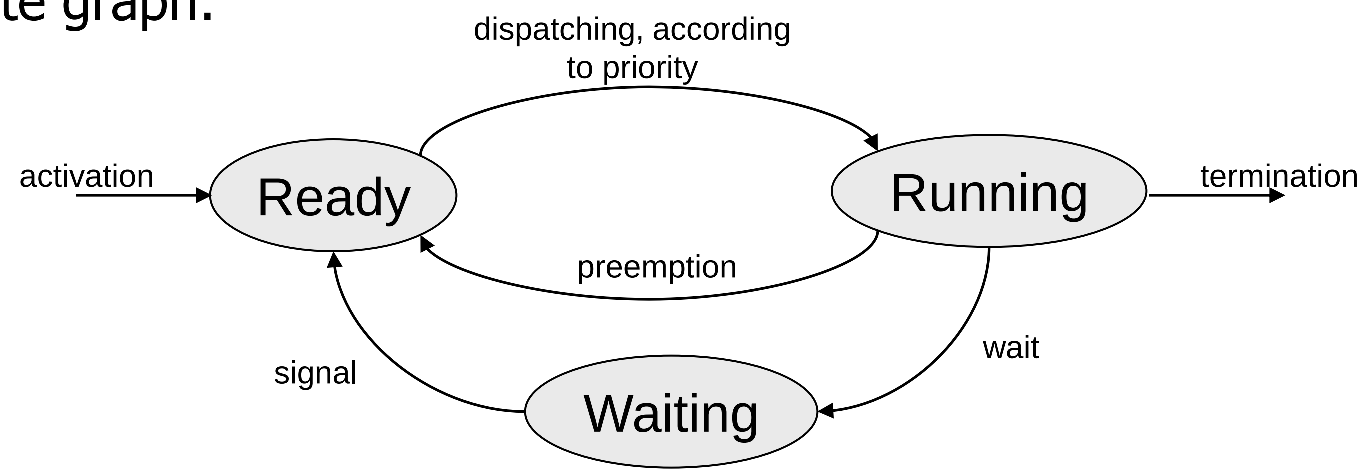
- Resources: data structures, files, devices, ...
  - private resource: dedicated to particular task
  - shared resource: available to more than one task

- To ensure consistency of shared resources, tasks must be granted exclusive access $\rightarrow$ mutually exclusive resources. Program sections during which exclusive access to a resource is required are called critical sections. A task waiting for a mutually exclusive resource is called blocked on that resource.

- Any task which needs to enter a critical section must wait until no other task is holding the resource. Otherwise the task enters the critical section and hold the resource. When the task leaves the critical section, the resource becomes free again.

Resource Access Protocols

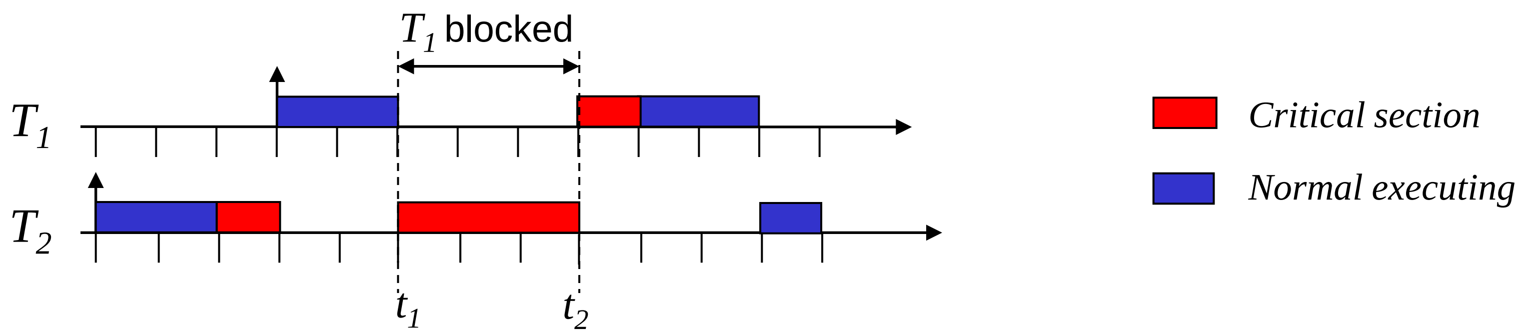
- Classical approach: Each mutually exclusive resource R_i is protected by a semaphore S_i. Each critical section on R_i must begin with $wait(S_i)$ and end with $signal(S_i)$ – the only operations supported on semaphores.
- Task state graph:



- Problem: Priority Inversion.

Priority Inversion

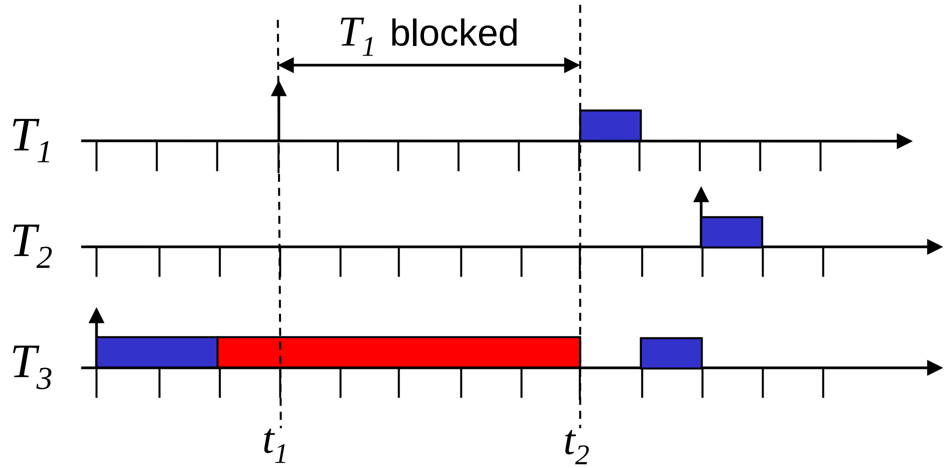
- Let two tasks T_1 and T_2 with priorities $P_1 > P_2$ be given that share a mutually exclusive resource R_k.



- T_2 is activated first, enters the critical section and locks the semaphore. When T_1 is released, it preempts T_2 since its priority is higher. However, when attempting to enter its critical section at t_1, T_1 is blocked on the semaphore, so T_2 resumes – although its priority is lower. In $[t_1,t_2]$ a priority inversion occurs.

Priority Inversion

- Naive solution: Disallow preemption during execution of critical sections.
  - May cause unnecessary blocking for a long period of time.
  - Example: Assume $P_1 > P_2 > P_3$. T_1 is blocked for a long time although it does not use any resource.



  - Better solutions required.

Priority Inheritance Protocol (PIP)

- Idea: modify the priority of the blocking tasks.
- Let J_i denotes a job, ie a generic instance of task T_i.
- When a job J_i blocks one or more higher-priority tasks, it temporarily inherits the highest priority of the blocked tasks. This prevents medium-priority tasks from preempting J_i.
- Let $\Gamma = \{T_i\}$ be a set of periodic tasks cooperating through M shared resources $R_1,\ldots,R_M$.
- Each resource R_i is guarded by a distinct semaphore S_i.
- Assume $d_i = \pi_i$ for all tasks T_i.
- The protocol can modify the priority of tasks. Thus:
  - nominal priority P_i
  - active priority p_i ($p_i \geq P_i$), which is dynamic and initially set to P_i.

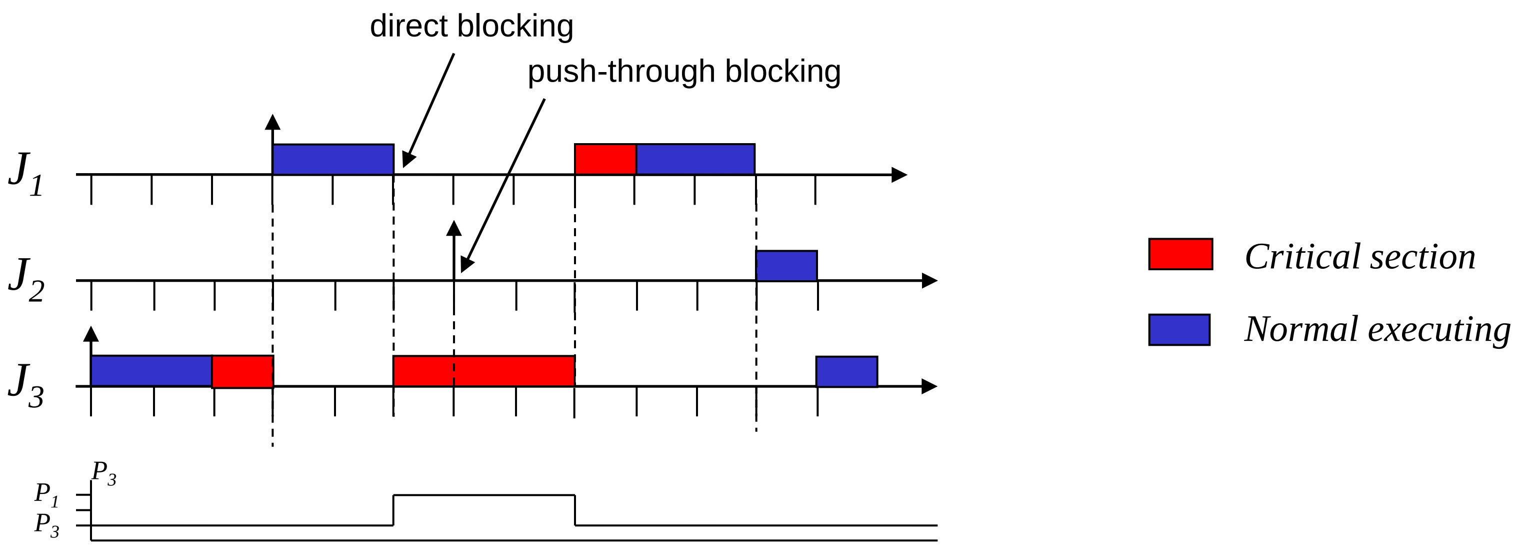
Priority Inheritance Protocol (PIP)

- Only one job at a time an be within the critical section corresponding to a particular semaphore S_i.
- Let $z_{i,j}$ denote the jth critical section of job J_i. The $S_{i,j}$ is the semaphore guarding $z_{i,j}$ and $R_{i,j}$ is the resource associated with $z_{i,j}$.
- Let $u_{i,j}$ denote the duration of $z_{i,j}$, ie the time needed by J_i to execute $z_{i,j}$ without interruption.
- We assume priority ordering for jobs $J_1, J_2,\ldots, J_n$ wrt nominal priorities such that $P_1 \geq P_2 \geq \ldots \geq P_n$.
- Critical sections are perfectly nested, ie either $z_{i,j} \subset z_{i,k}$ or $z_{i,k} \subset z_{i,j}$, or $z_{i,j} \cap z_{i,jk} = \emptyset$.

Definition of Priority Inheritance Protocol

- Jobs are scheduled based on active priorities.
- Jobs with the same priority are executed first come first served.
- When a job J_i tries to enter a critical section $z_{i,j}$ and resource $R_{i,j}$ is already held by a lower-priority job, J_i will be blocked. Otherwise J_i enters $z_{i,j}$.
- When a job J_i is blocked on a semaphore, it transmits its active priority to the job J_k that holds that semaphore. Then J_k resumes and executes the rest of its critical section with the inherited priority $p_k = p_i$.
- When J_k exits a critical section, it unlocks the semaphore and the highest-priority job blocked on that semaphore is awakened. The active priority of J_k is updated as follows: if no other jobs are blocked by J_k, p_k is set to its nominal priority P_k, otherwise it is set to the highest priority of the jobs blocked by J_k.
- Priority inheritance is transitive.

Priority Inheritance Protocol – Example



- Direct blocking: a high-priority job tries to acquire a resource held by a lower-priority job. Necessary to ensure consistency of shared resources.
- Push-through blocking: a medium-priority job is blocked by a lower-priority job that has inherited a higher priority from a job it directly blocks. Necessary to avoid unbounded priority inversion.

Priority Inheritance Protocol – Properties

- Lemma: If there are n lower-priority jobs that can block a job J_i, then J_i can be blocked for at most the duration of n critical sections (one for each of the n lower-priority jobs), regardless of the number of semaphores used by J_i.

- Proof:
  - A job J_i can be blocked by a lower-priority job J_k only if J_k has been preempted within a critical section $z_{k,j}$ and is still suspended in the moment when J_i is initiated.
  - Once J_k exits $z_{k,j}$, it can be preempted by J_i; thus J_i cannot be blocked by J_k again.
  - The same situation may happen for each of the n lower-priority jobs; therefore J_i can be blocked at most n times.

Priority Inheritance Protocol – Properties

- Lemma: If there are m distinct semaphores that can block a job J_i, then J_i can be blocked for at most the duration of m critical sections, one for each of the m semaphores.

- Proof:
  - Since semaphores are binary, only one of the lower-priority jobs J_k can be within a blocking critical section corresponding to a particular semaphore S_i.
  - Once S_i is unlocked, J_k can be preempted and can no longer block J_i. If all m semaphores that can block J_i are locked by m lower-priority jobs, then J_i can be blocked at most m times.

Priority Inheritance Protocol – Properties

- Theorem (Sha-Rajkumar-Lehoczky):
  Under the Priority Inheritance Protocol, a job J can be blocked for at most the duration of min(n,m) critical sections, where n is the number of low-priority jobs that could block J and m is the number of distinct semaphores that can be used to block J.

- Proof: immediately follows from the two previous lemmas.

PIP – Schedulability Analysis

- Liu/Layland: $$\sum_{i=1}^{N} \frac{c_i}{\pi_i} \le N(2^{\frac{1}{N}} - 1) \qquad (*)$$
- Let B_i be the maximum blocking time, due to lower-priority jobs, that a job J_i may experience.
- Theorem: A set of n periodic tasks using the Priority Inheritance Protocol can be scheduled by the Rate-Monotonic algorithm if
$$\forall 1 \le i \le n: \quad \sum_{k=1}^{i} \frac{c_k}{\pi_k} + \frac{B_i}{\pi_i} \le i(2^{\frac{1}{i}} - 1)$$
- Proof:
If the criterion holds then a job J_i has enough time even if it lasted for c_i+B_i, taking into account the preemption c_k/π_k from higher priority jobs.

PIP – Response Time Analysis

- To take resources into account, the blocking factor B_i must be added to the computation time of each task. This gives the following response time equation

$$R_i = c_i + B_i + I_i = c_i + B_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i}{\pi_j} \right\rceil c_j$$

- The corresponding recurrence equation is:

$$R_i^{(k+1)} = c_i + B_i + \left(\sum_{j=1}^{i-1} \left\lceil \frac{R_i^{(k)}}{\pi_j} \right\rceil c_j\right)$$

PIP – Computing the Blocking Time

- Let the ceiling $C(S_k)$ of a semaphore S_k be defined as

$$C(S_k) = \max\{ P_i \mid \text{job } J_i \text{ uses } S_k \}$$

- Let $D_{i,k}$ denote the duration of the longest critical section of task T_i among those guarded by semaphore S_k.
- Let a set of N periodic tasks that use M binary semaphores be given. Then the maximum blocking time B_i for each task T_i can be determined as follows:

$$B_i^l = \sum_{j=i+1}^{N} \max_k \{D_{j,k} | C(S_k) \geq P_\mathrm{i}\}$$

$$B_i^s = \sum_{k=1}^{M} \max_{j>i} \{D_{j,k} | C(S_k) \geq P_\mathrm{i}\}$$

$$B_i = \min(B_i^l, B_i^s)$$

PIP – Example

- Let a set of four tasks with three semaphores be given. The table shows the values $D_{i,k}$ for a job J_i and a semaphore S_k. The semaphore ceilings are given in parentheses.

$D_{i,k}$	$S_1(P_1)$	$S_2(P_1)$	$S_3(P_2)$
J_1	1	2	0
J_2	0	9	3
J_3	8	7	0
J_4	6	5	4

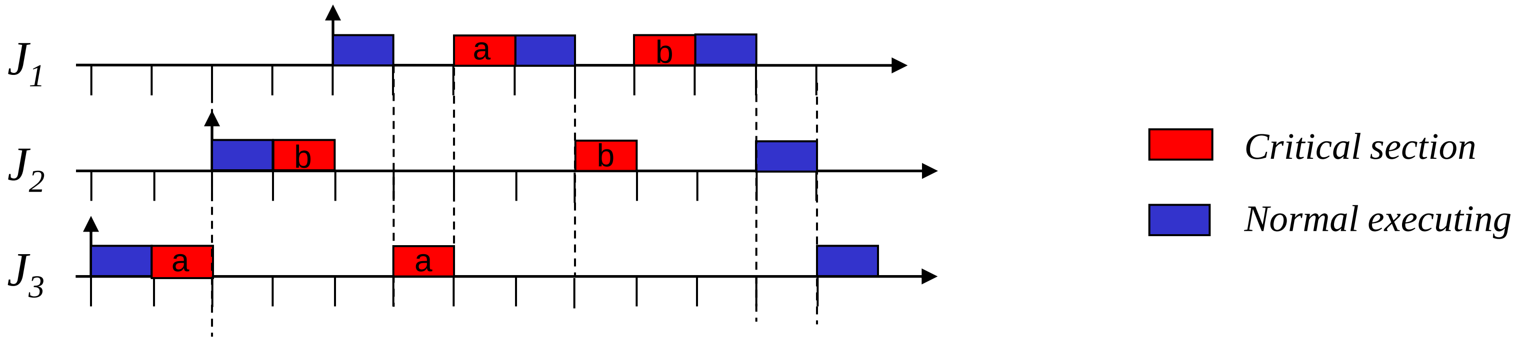
- Then the blocking factors for job J_1 are computed as follows:

$$B_1^l = \sum_{j=1+1}^{4} \max_k \{D_{j,k} | C(S_k) \geq P_1\} = 9 + 8 + 6 = 23$$

$$B_1^s = \sum_{k=1}^{M} \max_{j>1} \{D_{j,k} | C(S_k) \geq P_1\} = 8 + 9 = 17$$

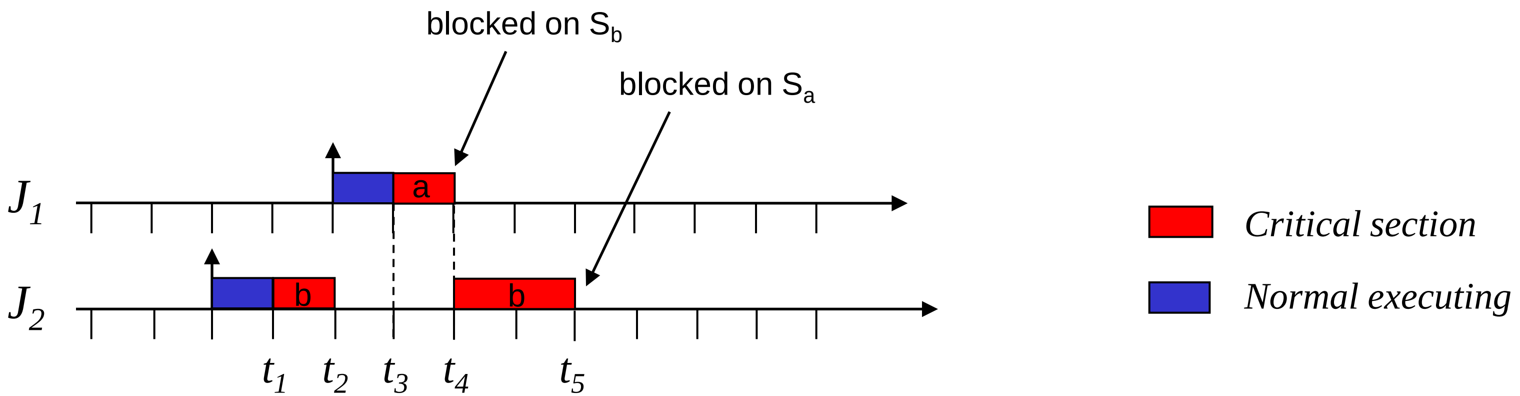
$$\Rightarrow B_1 = 17$$

PIP – Chained Blocking



- In the worst case, if J_1 accesses m distinct semaphores that have been locked by m lower-priority jobs, then J_1 will be blocked for the duration of m critical sections.

PIP – Deadlocks



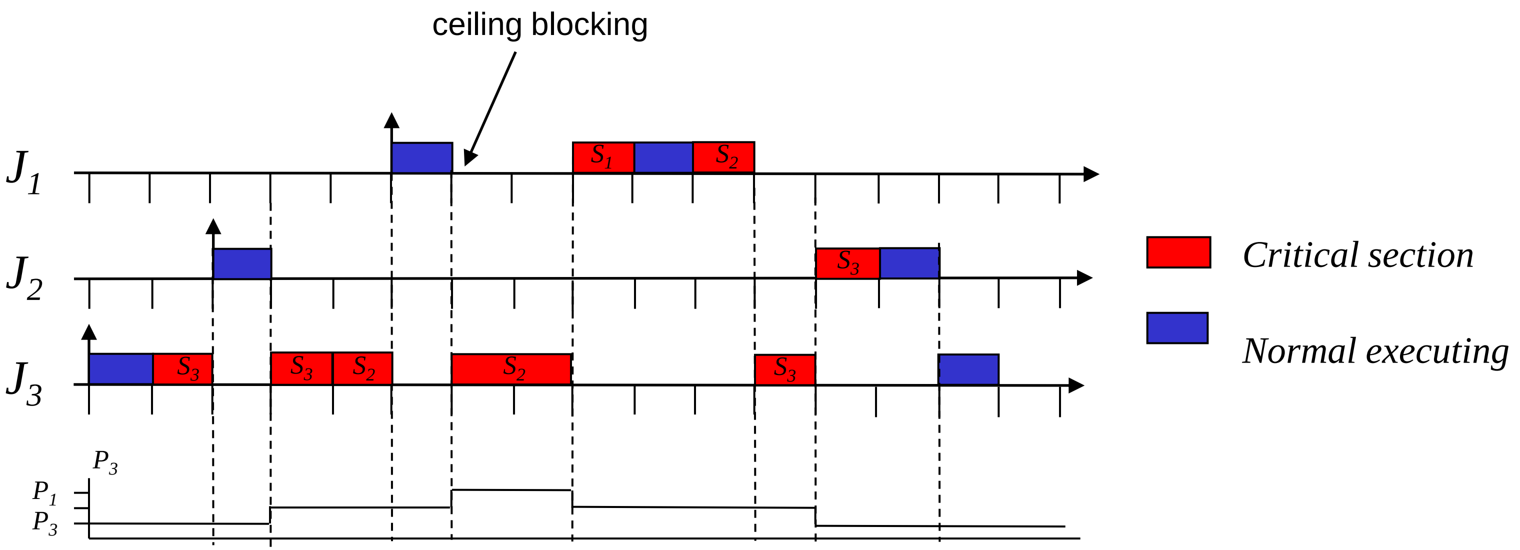
- t_1: J_2 locks S_b.
- t_2: J_2 is preempted by the higher-priority job J_1.
- t_3: J_1 locks S_a.
- t_4: J_1 is blocked on S_b. J_2 resumes and continues execution at the priority of J_1.
- t_5: J_2 attempts to lock S_a. => Deadlock!
- Note: deadlock is caused by erroneous use of semaphores.

The Priority Ceiling Protocol

- Each semaphore S_k is assigned a priority ceiling $C(S_k)$, equal to priority of the highest-priority task that can lock it. Note that $C(S_k)$ is a static value that can be computed offline.
- When a task T_i wants to lock a semaphore S_k, let H_i be the set of semaphores held by tasks different from T_i and $P^* = \max\{C(S') \mid S' \in H_i\}$.
- Task T_i gets the lock S_k only if $P_i > P^*$.
- Note that P^* is independent from the semaphore S_k.
- When a job J_i is blocked on a semaphore it transmits its priority to the job J_k that holds the semaphore. Hence, J_k resumes and executes the rest of its critical section with the priority of J_i. J_k is said to inherit the priority of J_i.
- When J_k exits a critical section, it unlocks the semaphore and the highest-priority job, if any, blocked on that semaphore is awakened. The active priority of J_k is set to the normal priority J_k if no other jobs are blocked by J_k, otherwise it is set to the highest priority of the jobs blocked by J_k.

The Priority Ceiling Protocol – Example

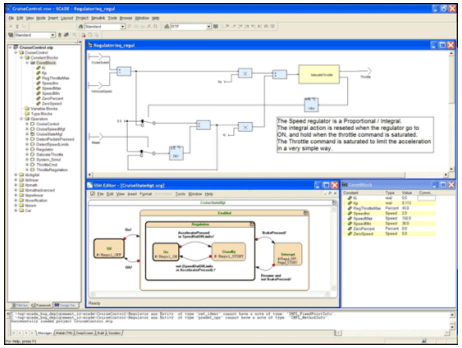
- Let three jobs J_1, J_2, and J_3 having decreasing priorities be given.
- J_1 sequentially accesses two critical sections guarded by semaphores S_1 and S_2.
- J_2 only accesses a critical section guarded by S_3.
- J_3 uses semaphore S_3 and then makes a nested access to S_2.
- This gives the following priority ceilings: $C(S_1)=P_1$, $C(S_2)= P_1$, $C(S_3)= P_2$.



The Priority Ceiling Protocol - Properties

- A high-priority process can be blocked at most once during its execution by any lower-priority process.
- Deadlocks are prevented.
- Transitive blocking is prevented.

Model-based Software Development



Esterel SCADE
- **SCADE language** ✓
- **SyncCharts** ✓

Generator

Lustre programs ✓

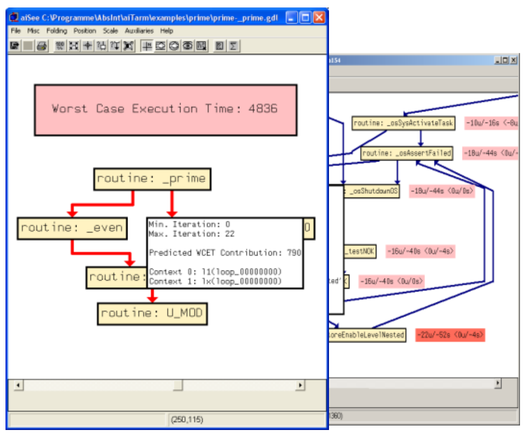
Esterel programs ✓

Compiler ✓

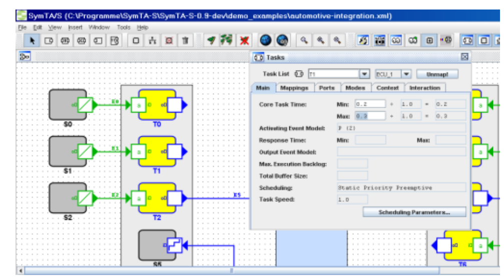
C Code

Compiler

Binary Code



aiT WCET Analyzer
- Timing Validation



SymTA/S
- System-level Scheduling & Schedulability Analysis