Lecture 13

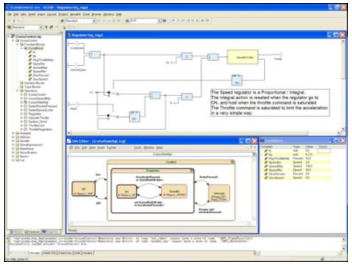
Real-Time Scheduling

Daniel Kästner
AbsInt GmbH

2013

Model-based Software Development

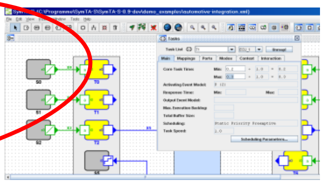
SCADE Suite



Application Model in SCADE (data flow + SSM) ✓

System Model (tasks, interrupts, buses, …)

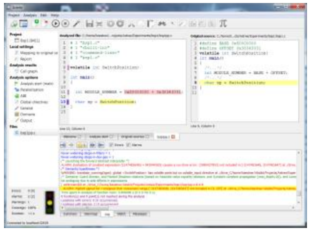
SymTA/S



System-level Schedulability Analysis

✓ Generator

Astrée



Runtime Error Analysis ✓

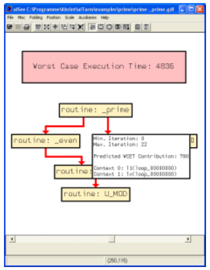
```
void Task (void)
{
   variable++;
   function();
   next++:
   if (next)
      do this;
   terminate()
}
```

C-Code

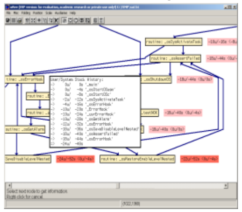
Compiler ✓

Binary Code

aiT



StackAnalyzer



Worst-Case Execution Time Analysis ✓

Stack Usage Analysis ✓

Setting the scene

- Hard real-time systems can be designed as a set of cooperating sequential processes (tasks).

- Questions:
  - In which order to execute tasks?
  - How to deal with shared resources?
  - How to guarantee timely execution?

The Endless Loop

```
Do forever
  request input device;
  fetch input value;
  do computation;
  request output device;
  write output;
End
```

The Basic Cyclic Executive

- Let three procedures A, B, and C be given.

```
Do forever
  call A;
  call B;
  call C;
End
```

The Time-Driven Cyclic Executive

- Let three procedures A, B, and C be given.

```
Do forever
  wait for timer interrupt;
  call A;
  call B;
  call C;
End
```

- The rate of hardware timer interrupts is the rate at which the procedures (tasks) must execute.

Multi-Rate Cyclic Executive

Task	Period	WCET
A	15	5
B	15	4
C	30	3
D	30	2
E	60	1

- Let the following task system be given:

```
Do forever // The major cycle
   wait for timer interrupt; //1st minor cycle
   A; B; C;
   wait for timer interrupt; //2nd minor cycle
   A; B; D; E;
   wait for timer interrupt; //3rd minor cycle
   A; B; C;
   wait for timer interrupt; //4th minor cycle
   A; B; D;
End
```

- Procedures are mapped onto a set of minor cycles that together constitute the complete schedule (or major cycle).

The Cyclic Executive

- Naive, but common way to implement concurrent hard real-time systems.
- No actual processes exist at run-time; each minor cycle is just a sequence of procedure calls
- Procedures share a common address space and can thus pass data between themselves. Concurrent access is not possible, thus no protection (e.g. semaphores) required.
- All process periods must be a multiple of the minor cycle time.

The Cyclic Executive

- Simple process modell:
  - Application consists of fixed set of processes
  - All processes are periodic
  - All processes are independent from each other
  - Context-switching times and other overhead is ignored
  - All processes have a deadline equal to their period
  - All processes have known worst-case execution time

The Cyclic Executive - Problems

- System is deterministic, but only fully so for the first task (at begin of major/minor cycle). All later tasks start to run whenever the preceding ones have ended.
- Hardware devices are polled. If they are not polled frequently enough, important events might be missed. If they are polled too frequently, processing power is wasted.
- Difficult to incorporate processes with long periods.
- If procedures are split up to form tasks with lower execution times, finding the right granularity of “processes” is difficult.
- Code for logically independent tasks is interleaved.
- Sporadic activities cannot be incorporated.
- Difficult to construct (NP complete) and difficult to maintain.

The Scheduling Problem: Classification

- Scheduling problems usually are classified according to a set of criteria:
  - the cost function
  - hard deadlines vs. soft deadlines
  - periodic vs. aperiodic vs. sporadic events
  - preemptive vs. non-preemptive
  - static vs. dynamic
  - online vs. offline

The Scheduling Problem: Classification

- Tasks which must be executed once every *p* units of time are called periodic, and *p* is called their period. Each execution of a periodic task is called a job.
- Tasks which are not periodic are called aperiodic.
- Aperiodic tasks requesting the processor at unpredictable times are called sporadic, if there is a minimum separation between the times at which they request the processor.

- A preemptive scheduler can arbitrarily suspend a process's execution and restart it later without affecting the functional behavior of the process. Preemption typically occurs when a higher priority process becomes runnable. Non-preemptive schedulers do not suspend processes in this way.

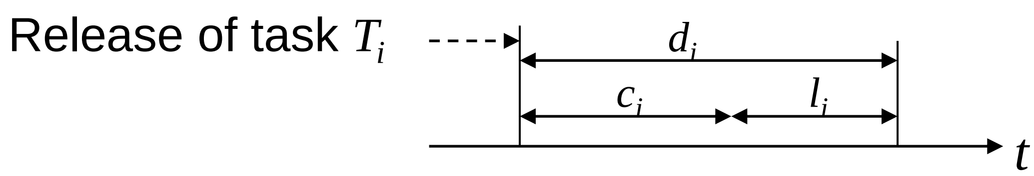
The Scheduling Problem: Classification

- An offline scheduling algorithm makes all scheduling decisions prior to the running of the system. Online scheduling algorithms schedule tasks at run-time; they can be either static or dynamic.

- In a static scheduling algorithm calculating the schedules is based on a process's characteristics available before the system is run. It requires little runtime overhead.
- A dynamic method schedules at run-time, taking into account both process characteristics and the current state of the system. It has higher run-time cost but can deal with non-predicted events and can give greater processor utilization.

The Task Model

- Let $\Gamma = \{ T_i \}$ be a set of tasks. Then let
  - r_i be the release time (or arrival time) which is the time at which T_i is ready for processing
  - c_i be the worst-case execution time of T_i
  - d_i be the deadline interval, ie the time between T_i becoming available and the time until which T_i has to finish execution
  - $l_i = d_i$-c_i be the laxity or slack of T_i.
  - In $\{ T_i \}$ precedence constraints among tasks may be defined. $T_i \rightarrow T_j$ means that the processing of T_i must be completed before T_j can be started.



Task Model

- The following parameters can be calculated from a given schedule:
  - Completion Time C_i
  - Response Time $R_i = C_i - r_i$
  - Lateness $L_i = C_i - d_i$
  - Tardiness $D_i = max \{ C_i - d_i , 0 \}$
- Some performance measures / goal functions:
  - Schedule Length (makespan) $C_{max} = max\{ C_i \}$
  - Maximum Lateness $L_{max} = max\{ L_i \}$
- Critical instant: That time at which the release of a task will produce the largest response time.

- Scheduling to minimize the makespan with release times and deadlines is NP hard.

Overview

- Static-Priority Scheduling (Fixed-priority Scheduling)
- Dynamic-Priority Scheduling
- Schedulability and Response Time Analysis
- Further reading:
  - Giorgio Buttazzo. *Hard Real-Time Computing Systems. Predictable Scheduling Algorithms and Applications.* 2nd Edition. Springer, 2005.
  - Jane Liu. *Real-Time Systems*. Prentice Hall, 2000.

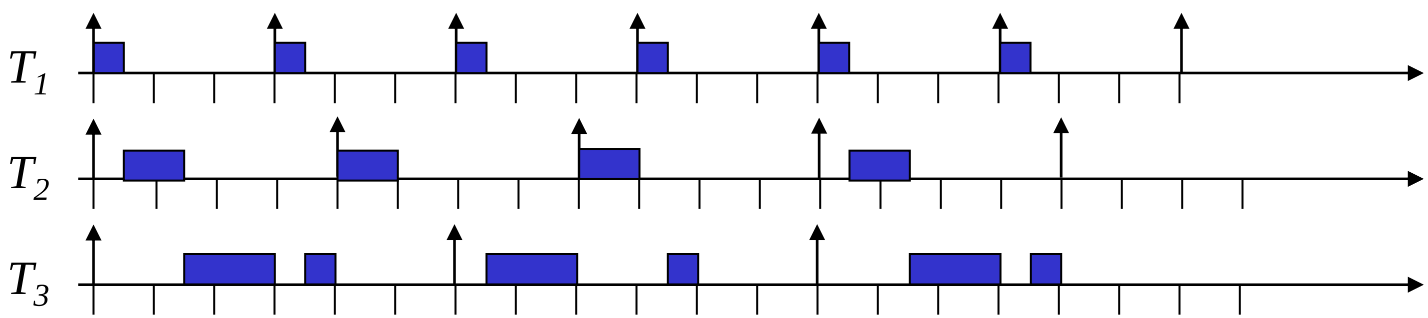
Fixed-Priority Scheduling

- Under fixed-priority scheduling, different jobs of a task are assigned the same priority.

- A fixed-priority scheduling scheme S is optimal if the following criterion is satisfied:
  If any process can be scheduled with some fixed-priority assignment scheme,
  then the given process can also be scheduled with scheme S.

Rate Monotonic Scheduling

- Let each process have a unique priority P_i based on its period π_i.
- We assume that the shorter the period, the higher the priority, ie $\pi_i < \pi_j \Leftrightarrow P_i > P_j$.
- Further assume $d_i = \pi_i$ for all tasks T_i.
- Example schedule: T_1 with $\pi_1=3$ and $c_1=0.5$, T_2 with $\pi_2=4$ and $c_2=1$ and T_3 with $\pi_3=6$ and $c_3=2$.



Rate Monotonic Scheduling

- The priority of a process is derived from its temporal requirements, not its importance to the system, nor its integrity.
- Note: priority 1 is lowest (least) priority.

Task	Period π	Priority P
A	15	5
B	30	4
C	100	1
D	45	2
E	35	3

- The schedulability depends on the period and the maximal computational requirements of each process.

Processor Utilization

- Let $\Gamma = \{T_i\}$ be a set of tasks.
  The utilization U of a task set is defined as $U = \sum_{i=1}^{N} \frac{c_i}{\pi_i}$
- Corollary: If the utilization factor of a task set $\Gamma = \{ T_i \}_{i=1..N}$ is greater than one, the task set cannot be scheduled by any algorithm.
- PROOF: Let $\Pi = \pi_1 \pi_2 \ldots \pi_N$ be the product of all periods.
  If $U > 1$, then also $U\,\Pi > \Pi$, which can be written as:

$$\sum_{i=1}^{N} \frac{\Pi}{\pi_i} c_i > \Pi$$

  - Π/π_i is the number of times task T_i is executed in the interval Π.
  - $(\Pi/\pi_i)c_i$ is the total computation time requested by T_i in the interval Π.
  - Thus: if the total demand in computation time is higher than the available processor time, there can be no feasible schedule for the task set. ■

Processor Utilization

- There exists a maximum value of U below which Γ is schedulable and above which Γ is not schedulable. This limit depends on
  - the task set, ie. the relations among task's periods
  - and on the algorithm used to schedule the tasks.
- Let $U_{ub}(\Gamma,A)$ be this upper bound of the processor utilization factor for a task set Γ under an algorithm *A*.
- When $U=U_{ub}(\Gamma,A)$, Γ fully utilizes the processor. Then Γ is schedulable but an increase in computation time in any of the tasks will make the set infeasible.
- For a given algorithm A, the least upper bound $U_{lub}(A)$ is the minimum of the utilization factors over all task sets that fully utilize the processor:

$$U_{\text{lub}}(A) = \min_{\Gamma} U_{ub}(\Gamma, A)$$

- Any task set whose processor utilization factor is below $U_{lub}(A)$ is schedulable by A $\Rightarrow$ With U_{lub} schedulability can be easily verified!

Rate Monotonic Scheduling

- Theorem [Liu and Layland]: A system of N independent, preemptable periodic tasks T_i with $d_i = \pi_i$ can be feasibly scheduled on a processor according to the rate monotonic algorithm if its total utilization U is at most

$$U_{RM} = N(2^{\frac{1}{N}} - 1)$$

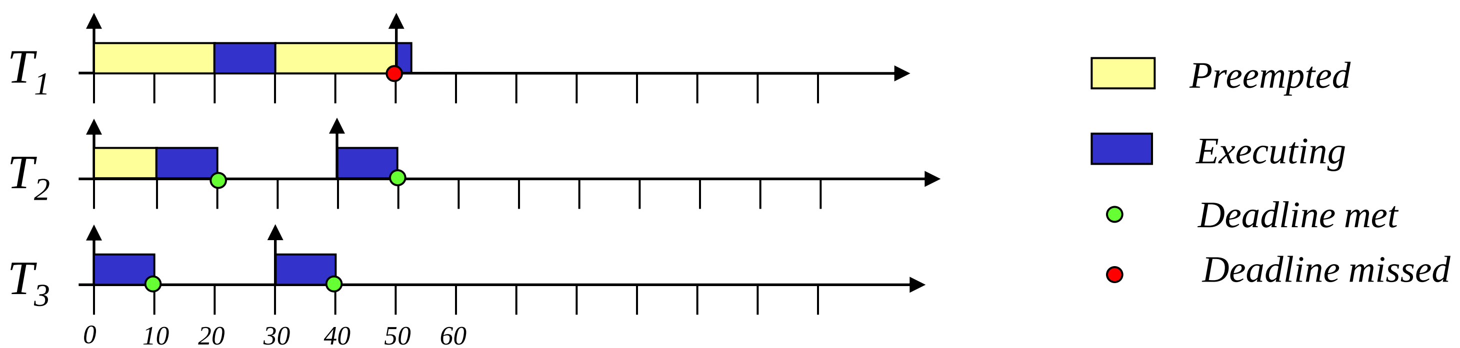
- Note: U_{RM} asymtotically approaches $ln2$ (69.3%).

N	$U_{RM}(N)$
1	1
2	0.828
3	0.779
4	0.756
5	0.743
6	0.734

Example: Process Set A

Process	Period π	WCET c	Priority P	Utilization U
T1	50	12	1	0.240
T2	40	10	2	0.250
T3	30	10	3	0.333

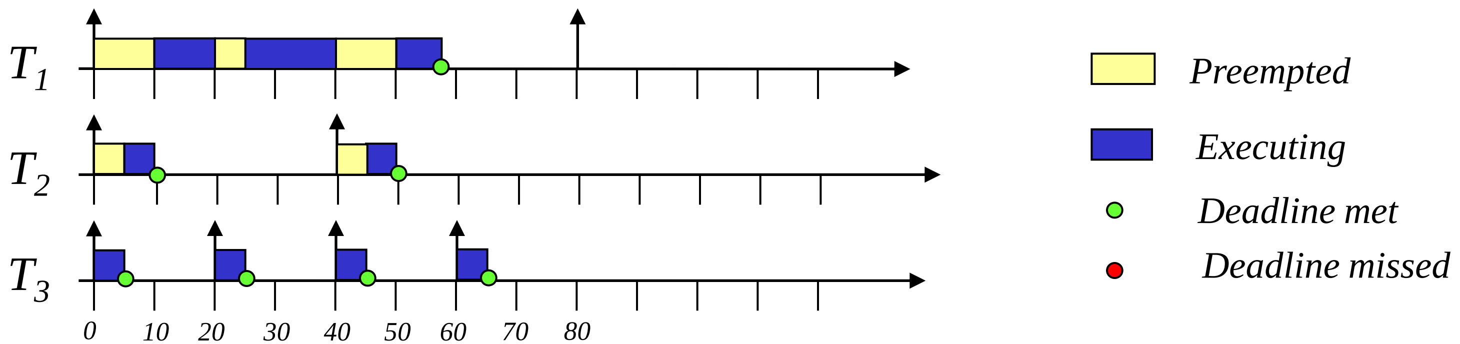
- The combined utilization is U=12/50+10/40+10/30=0.823.
- Since this is above the threshold for three processes ($U_{RM}(3)=0.78$), this process set fails the utilization test.



Example: Process Set B

Process	Period π	WCET c	Priority P	Utilization U
T1	80	32	1	0.400
T2	40	5	2	0.125
T3	20	5	3	0.250

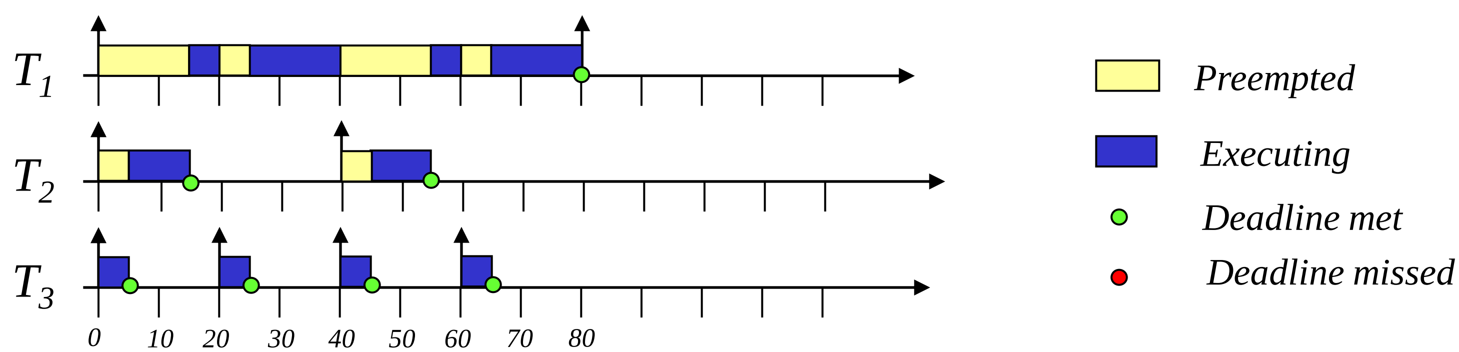
- The combined utilization is U=32/80+5/40+5/20=0.775.
- Since this is below the threshold for three processes ($U_{RM}(3)=0.78$), this process set will meet all its deadlines.



Example: Process Set C

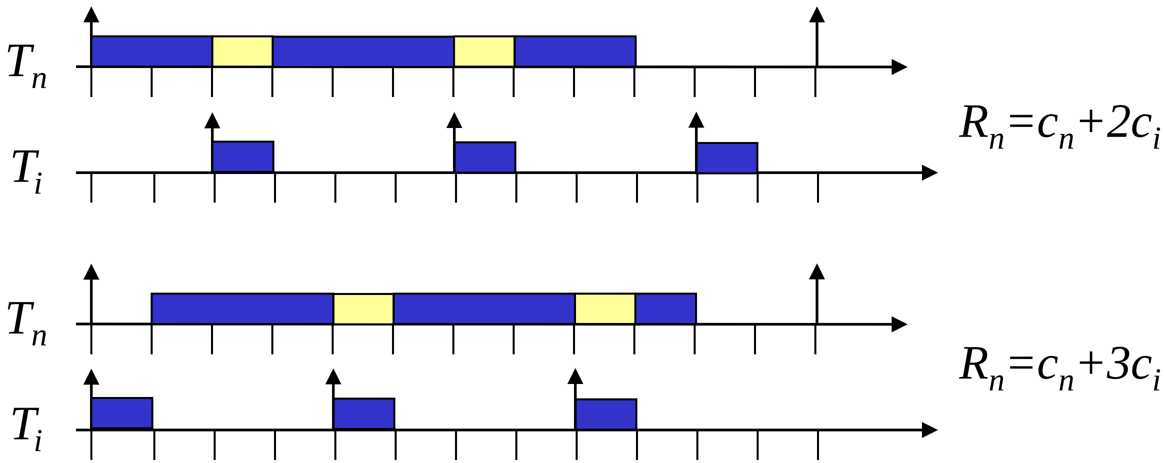
Process	Period π	WCET c	Priority P	Utilization U
A	80	40	1	0.500
B	40	10	2	0.250
C	20	5	3	0.250

- The combined utilization is 1.0.
- Since this is above the threshold for three processes ($U_{RM}(3)=0.78$), this process set fails the utilization test. Nevertheless the process set will meet all its deadlines.



Critical Instants

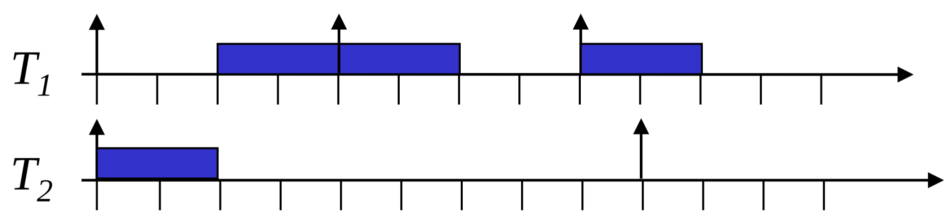
- Corollary: A critical instant for a task occurs whenever the task is released simultaneously with all higher-priority tasks.
- Let $\Gamma = \{ T_i \}_{i=1..N}$ be a set of periodic tasks, ordered by increasing periods, ie $\pi_1 < \pi_2 < \ldots < \pi_n$ and, thus $P_1 > P_2 > \ldots > P_N$ and let $n > i$.



- Intuition:
  - The response time of task T_n is delayed by the interference of a task T_i with higher priority.
  - Advancing the release time of T_i may increase the completion time of T_n.

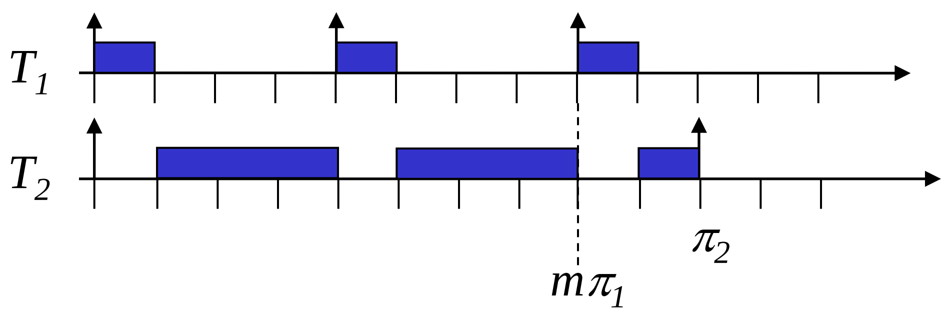
Optimality of Rate Monotonic Scheduling

- Observation: If all tasks are feasible at their critical instants, then the task set is schedulable in any other condition.
- Theorem: If a task set is schedulable by an arbitrary fixed priority assignment, then it is also schedulable by RM.
- PROOF:
  - Let T_1 and T_2 be two periodic tasks with $\pi_1 < \pi_2$. Assume that their priorities are not assigned according to RM, ie $P_2 > P_1$.
  - At a critical instant, the schedule is feasible if the following inequality is satisfied: $c_1 + c_2 \leq \pi_1$ $(Eq.\ 1)$

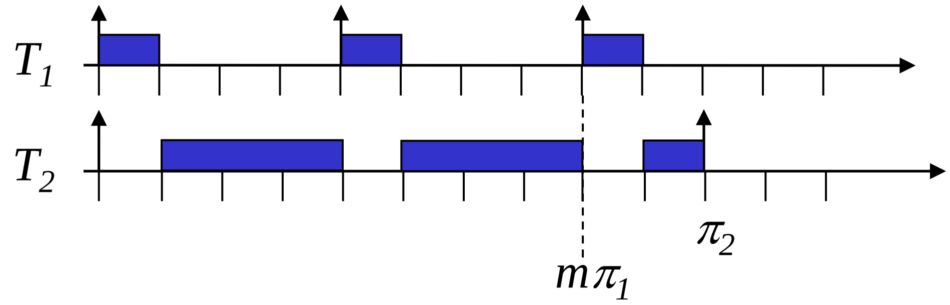


Optimality of Rate Monotonic Scheduling

- Now we want to show that T_1 and T_2 are also schedulable with the RM priority scheme, ie when $P_1 > P_2$.
- Let $m = \left\lfloor \frac{\pi_2}{\pi_1} \right\rfloor$ be the number of periods of T_1 entirely contained in π_2.
- Then two cases have to be distinguished:
- Case 1: The computation time c_1 is short enough that all requests of T_1 within the critical time zone of T_2 are completed before the second request of T_2. That is: $c_1 \le \pi_2 - m\pi_1$



Optimality of Rate Monotonic Scheduling



- Then the task set is schedulable if

$$(m+1)c_1 + c_2 \le \pi_2 \qquad (Eq.\ 2)$$

- We have to show that Eq.1 $\Rightarrow$ Eq.2.

$$c_1 + c_2 \le \pi_1 \qquad (Eq.\ 1)$$

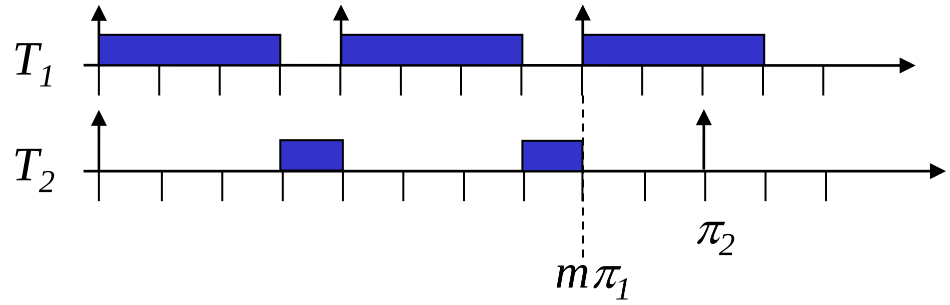
$$\Leftrightarrow \quad mc_1 + mc_2 \le m\pi_1$$

$$\Leftrightarrow \quad mc_1 + c_2 \le mc_1 + mc_2 \le m\pi_1, \text{since } m \ge 1$$

$$\Leftrightarrow \quad (m+1)c_1 + c_2 \le m\pi_1 + c_1$$

$$\Leftrightarrow \quad (m+1)c_1 + c_2 \le m\pi_1 + c_1 \le \pi_2, \text{since } c_1 \le \pi_2 - m\pi_1$$

Optimality of Rate Monotonic Scheduling



- Case 2: The execution of the last request of T_1 in the critical time zone of T_2 overlaps the second request of T_2. That is:

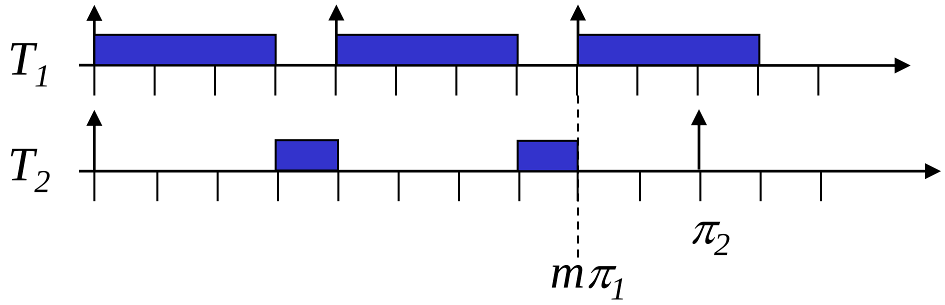
$$c_1 > \pi_2 - m\pi_1$$

- Then the task set obviously is schedulable, if

$$mc_1 + c_2 \leq m\pi_1 \qquad (Eq.\ 3)$$

- We have to show that Eq.1 $\Rightarrow$ Eq.3.

Optimality of Rate Monotonic Scheduling



- Consider again Eq. 1.

$$c_1 + c_2 \le \pi_1 \qquad (Eq.\ 1)$$

$$\Leftrightarrow \qquad mc_1 + mc_2 \le m\pi_1$$

$$\Leftrightarrow \qquad mc_1 + c_2 \le mc_1 + mc_2 \le m\pi_1, \text{since } m \ge 1$$

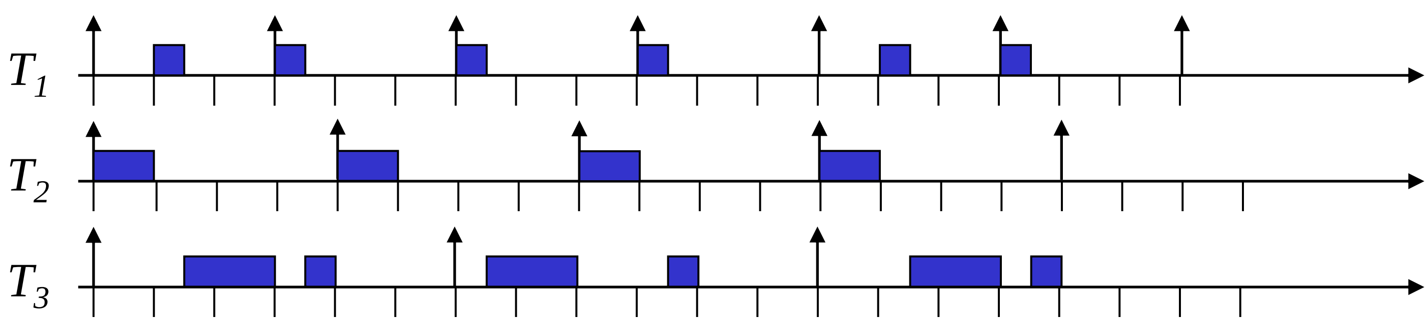
- This directly shows

$$mc_1 + c_2 \le m\pi_1 \qquad (Eq.\ 3)$$

■

Deadline Monotonic Scheduling

- Let each process have a unique priority P_i based on its relative deadline d_i.
- Same as rate monotonic, if each task's relative deadline equals its period.
- We assume that the shorter the deadline, the higher the priority, ie $d_i < d_j \Leftrightarrow P_i > P_j$.
- Example schedule: T_1 with $\pi_1 = d_1=3$ and $c_1=0.5$, T_2 with $\pi_2=4$, $d_2=2$ and $c_2=1$ and T_3 with $\pi_3=d_3=6$ and $c_3=2$.



Schedulability Analysis

- The rate monotonic schedulability test can be applied also to deadline monotonic scheduling, by reducing periods to relative deadlines:

$$\sum_{i=1}^{N} \frac{c_i}{d_i} \leq N(2^{\frac{1}{N}} - 1)$$

- However, this test significantly overestimates the workload on the processor.
- Observations:
  - The worst-case processor demand occurs when all tasks are released at their critical instants.
  - For each task T_i the sum of its processing time and the interference (preemption) imposed by higher priority tasks must be less than or equal to its deadline d_i.

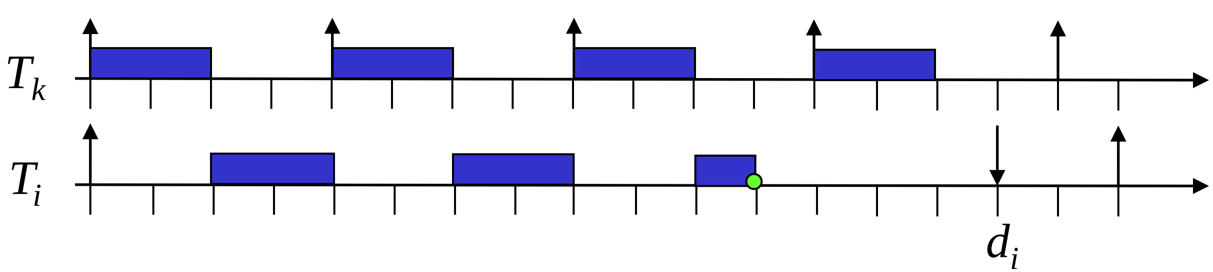
Schedulability Analysis

- Assume that tasks are ordered by increasing relative deadlines: $i<j \Leftrightarrow d_i<d_j \Leftrightarrow P_i>P_j$.
- Then a task set $\Gamma = \{ T_i \}_{i=1..N}$ is schedulable if the following condition is satisfied:

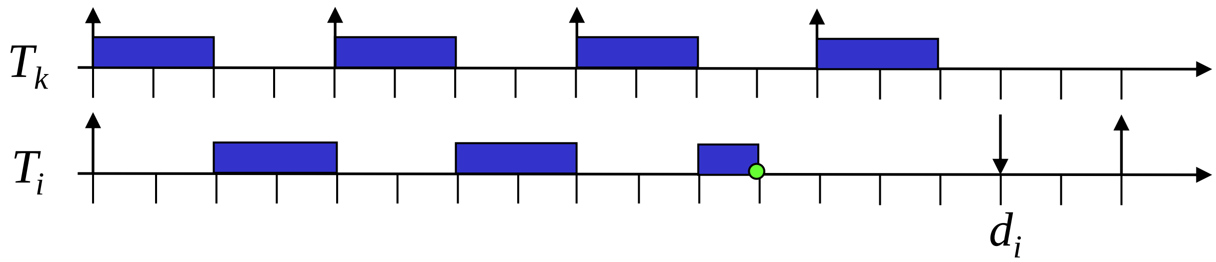
$$\forall 1 \le i \le N : c_i + I_i \le d_i$$

- where I_i is a measure of the interference of T_i, which can be computed as the sum of the processing times of all higher-priority tasks released before d_i:

$$I_i = \sum_{j=1}^{i-1} \left\lceil \frac{d_i}{\pi_j} \right\rceil c_j$$



Schedulability Analysis



- Note that this test is sufficient but not necessary.
- I_i is calculated by assuming that each higher-priority task exactly interferes $\left\lceil \frac{d_i}{\pi_j} \right\rceil$ times during the execution time of T_i. However, since T_i may terminate earlier, the actual interference may be smaller.
- A sufficient and necessary schedulability test for DM must take the exact interleaving of higher-priority tasks into account for each process.