1 LR(0) and SLR(1)

a) The following two derivations violate the LR(0) property:

\[ S' \Rightarrow^* rm \ bBb \Rightarrow_{rm} bcb \]
\[ S' \Rightarrow_{rm} bC \Rightarrow_{rm} bc \]

and \( 0 : b = 0 : \varepsilon \leftrightarrow \varepsilon = \varepsilon \), but \( B \neq C \).

b) LR-DFA(\( G \)):
where the states $S_0$ to $S_{12}$ are defined as follows:

$$S_0 = \{ \{ S' \rightarrow .S \} , \{ S \rightarrow .bA \} , \{ S \rightarrow .c \} \}$$

$$S_1 = \{ \{ S' \rightarrow S \} \}$$

$$S_2 = \{ \{ S \rightarrow .b.A \} , \{ A \rightarrow .Bb \} , \{ A \rightarrow .abS \} , \{ A \rightarrow .C \} , \{ B \rightarrow .b \} , \{ B \rightarrow .c \} , \{ C \rightarrow .c \} \}$$

$$S_3 = \{ \{ S \rightarrow .c. \} \}$$

$$S_4 = \{ \{ A \rightarrow a.bS \} \}$$

$$S_5 = \{ \{ B \rightarrow .b \} \}$$

$$S_6 = \{ \{ B \rightarrow .c. \} , \{ C \rightarrow .c. \} \}$$

$$S_7 = \{ \{ S \rightarrow b.A. \} \}$$

$$S_8 = \{ \{ A \rightarrow B.b \} \}$$

$$S_9 = \{ \{ A \rightarrow C. \} \}$$

$$S_{10} = \{ \{ S' \rightarrow S. \} \}$$

$$S_{11} = \{ \{ A \rightarrow abS. \} \}$$

$$S_{12} = \{ \{ A \rightarrow Bb. \} \}$$

c) State $S_6$ is the inadequate state of the LR-DFA($G$). The SLR(1) lookahead sets for each complete item of $[X \rightarrow \alpha.]$ of the state $S_6$ are computed as follows:

$$FOLLOW_1(S') = \{ \# \}$$

$$FOLLOW_1(S) = FOLLOW_1(S') \cup FOLLOW_1(A)$$

$$FOLLOW_1(A) = FOLLOW_1(S)$$

$$FOLLOW_1(B) = \{ b \}$$

$$FOLLOW_1(C) = FOLLOW_1(A)$$

Fixpoint iteration provides the SLR(1) lookahead sets:

$$S' \quad S \quad A \quad B \quad C$$

\{\#\} \quad \emptyset \quad \emptyset \quad \{ b \} \quad \emptyset
\{\#\} \quad \{\#\} \quad \{\#\} \quad \{ b \} \quad \{\#\}
\{\#\} \quad \{\#\} \quad \{\#\} \quad \{ b \} \quad \{\#\}

After adding the lookahead sets, the state $S_6$ contains the items:

$$S_6 = \{ \{ B \rightarrow .c. , \{ b \} \} , \{ C \rightarrow .c. , \{\#\} \} \}$$

All conflicts are resolved, because $\{ b \} \cap \{\#\} = \emptyset$.

2 LR(0) Grammars

a) Assume all regular languages are LR(0). Then also $L = \{ a, aa \}$ must LR(0) and there must be a LR(0) grammar $G$ producing $L$ and a corresponding parser. However after the consumption of the first $a$ such a parser either shifts (to read the next $a$) or reduces (to finish parsing). Since
there is no lookahead, the parser can not decide which action to take and can not resolve the shift-reduce conflict. Therefore, the assumption is wrong and there are regular languages which are not LR(0).

b) The language \( L = \{ a^n b^n | n > 0 \} \) is LR(0) since it is produced by the following LR(0) grammar:

\[
S' \rightarrow S \\
S \rightarrow AB | ASB \\
A \rightarrow a \\
B \rightarrow b
\]

As shown in the previous exercise, \( L \) is not regular. Therefore not all LR(0) languages are regular.

3 Attribute Grammars

a) Dependency graph for the word \textit{abb}:

\[
\begin{align*}
S & \rightarrow \phi(s) \\
A_1 & \rightarrow \phi(A_1.s) \\
A_2 & \rightarrow A_2.s \\
B_2 & \rightarrow B_2.s \\
A_3 & \rightarrow A_3.s \\
B_3 & \rightarrow B_3.s
\end{align*}
\]

b) Equation system:

\[
\begin{align*}
A_1.i &= \phi(A_1.s) & A_1.s &= A_2.s \\
A_2.i &= B_2.s & A_2.s &= A_3.s \\
B_2.i &= A_1.i & B_2.s &= B_2.i \\
A_3.i &= B_3.s & A_3.s &= A_3.i \\
B_3.i &= A_2.i & B_3.s &= B_3.i \\
\end{align*}
\]

Thus \( A_1.i = \phi(A_1.i) \).

c) The system has exactly one solution for \( \phi(x) = c \) for any \( c \in \mathbb{N} \). The equation system has infinitely many solution for \( \phi(x) = x \).