1 Finite State Automata 8+2 Points

a) For the regular expression \( r = b(a((ba)c)^*) \) over the alphabet \( \Sigma = \{a, b, c\} \), construct a nondeterministic finite automaton, a deterministic finite automaton and a minimal deterministic finite automaton following the construction rules from lecture.

b) The slides provide rules for constructing nondeterministic finite automata for the regular expressions \( r, r_1| r_2 \) and \( r^* \). Define a rule for \( r^+ = rr^* \), such that the resulting automaton has less states than the one for \( rr^* \).

2 Greed 5+5 Points

All lex-like scanners scan the input greedily. Can this lead to complications?

a) Provide an automaton and a corresponding input word, such that the maximal munch strategy has runtime \( \Omega(n^2) \).

b) Does there exist a sequence of regular expressions \( \alpha_1, \ldots, \alpha_n \) over \( \Sigma \) and a word \( w \in \Sigma^* \), such that:

1. there exist \( w_1, \ldots, w_k \in \Sigma^* \) and \( t_1, \ldots, t_k \in \{1, \ldots, n\} \), such that \( w = w_1 \ldots w_k \) and \( w_i \in \llbracket \alpha_{t_i} \rrbracket \) (that is, there is a match of the complete word), and
2. for each such dissection there exist \( j \in \{1, \ldots, k - 1\}, x, y \in \Sigma^+ \) and \( s \in \{1, \ldots, n\} \), such that \( w_{j+1} \ldots w_k = xy \) and \( w_jx \in \llbracket \alpha_s \rrbracket \) (that is, greedy scanning does not take us to a match)?

\(^1\)For a regular expression \( e \), \( \llbracket e \rrbracket \) is the language defined by \( e \)
Let the context free grammar $G = (\{S, L, M, I, E, B\}, \{a, b, c, d, e, f, g, h, i\}, P, S)$, with the productions $P$ given as:

$$
S \rightarrow L \mid gMB \mid \varepsilon \\
L \rightarrow bd \mid cd \mid iIE \mid d \\
M \rightarrow L \mid LS \mid \varepsilon \\
I \rightarrow eafL \mid eafgMB \\
E \rightarrow M \mid gMB \mid \varepsilon \\
B \rightarrow h
$$

Give a succeeding run of the non-deterministic pushdown automaton constructed by the algorithm from the lecture for $G$ on the input word $ieafgbdcdhbdcd$.\footnote{You do not need to explicitly write down the set of all rules.}