Register Allocation for SSA Form Programs

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Joint work started at Karlsruhe

26.04.2007

Outline

Register Allocation

- Register Allocation is the task of mapping the program's variables to processor registers
- Issues to be covered:

Spilling	Put variables into memory if there are not enough registers
Coalescing	Eliminate unnecessary copies in the program

- Often reduced to graph coloring

Liveness

Definition

A variable v is live at a label ℓ if there is a path from ℓ to a use of v not containing a definition of v.

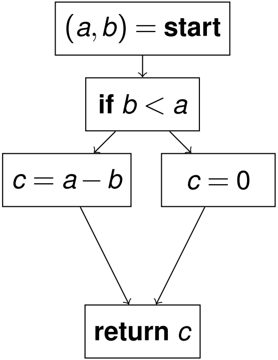
Definition

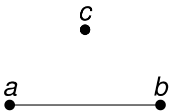
The live-range of a variable v are the labels where v is live.

- Conservative approximation by dataflow analyses

Interference Graphs

- Two variables interfere if they are live at the same label
- Each variable corresponds to a node in the interference graph (IG)
- Whenever two variables interfere, there is an edge between the corresponding nodes





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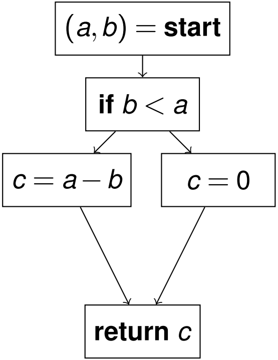
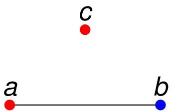




Coloring gives register allocation

Spilling

- Register need larger than number of registers ($\chi(IG) > k$)
- Determine subset of (sub-)live-ranges
- Rewrite program, inserting spills and reloads to temporarily store values in memory
- Assign storage locations for spilled values

Coalescing

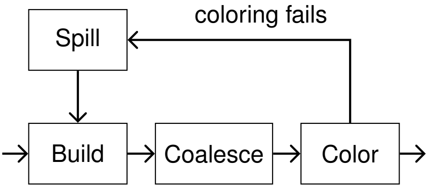
- In some phases compilers insert copy instructions to
  - implement operations by code sequences (correctness)
  - simplify a translation step (engeneering)
- Coalescing removes copy instructions by joining live-ranges
- Joining means assigning the same register to the source and target

`mov ebx, [esi]`
`add ebx, ebx`
`mov eax, ebx`
`mul ecx`
	$\Rightarrow$	`mov eax, [esi]`
		`add eax, eax`
		
		`mul ecx`

- More general: reduce useless value movement

Outline

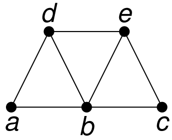
Chaitin/Briggs Register Allocator



- Every undirected graph can occur as an interference graph
- Determining chromatic number is $\mathscr{NP}$-complete
- Color using heuristic $\Rightarrow$ Iteration necessary on failure

Coloring

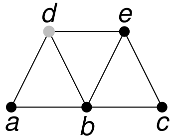
- Subsequently remove the nodes from the graph (simplify)



elimination order

Coloring

- Subsequently remove the nodes from the graph (simplify)



elimination order
d,

Coloring

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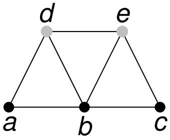

elimination order
d, e,

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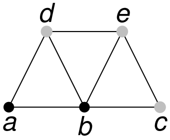

elimination order
d, e, c,

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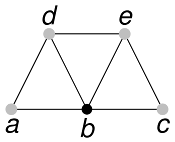

elimination order
d, e, c, a,

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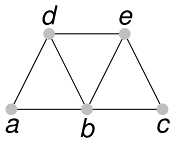

elimination order
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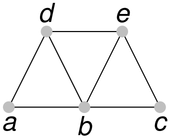
Coloring

- Subsequently remove the nodes from the graph (simplify)
- Re-insert the nodes in reverse order
- Assign each node the next possible color



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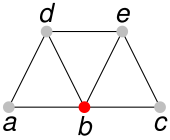

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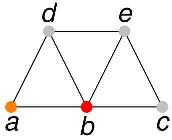

elimination order
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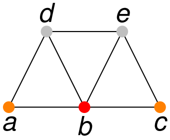

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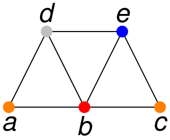

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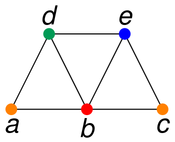


elimination order

Spilling

- Nodes with less then *k* neighbors can always be colored
- Simplify phase removes those nodes
- Other nodes get spilled pessimistically

Or:

- Other nodes are optimistically removed as well
- Hope for a free color despite many neigbors
- Delays spill decision until actual color assignment

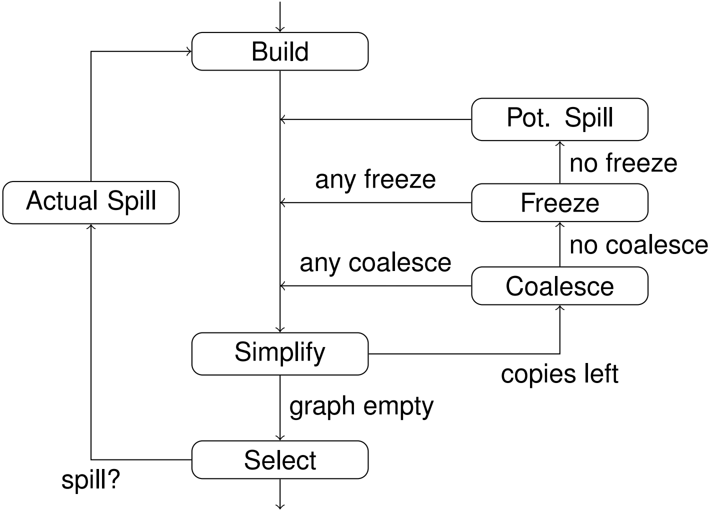
Inserting spill code invalidates liveness information
$\Rightarrow$ restart by rebuilding the IG

Coalescing

- Coalescing merges nodes in the graph to force both values into the same register
- Less but longer live ranges
- Problem: Coalescing influences colorability

- First heuristic ignored negative effects (aggressive)
- Later approaches restricted coalescing (conservative/iterated)
- State of the art has undo-capabilities (optimistic)

“Iterated” Allocator (Appel & George 96)



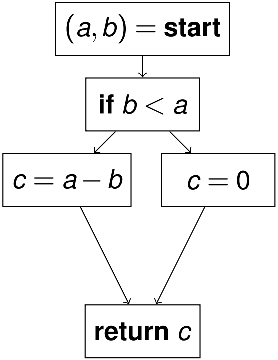
Outline

Outline

Static Single Assignment Form

- Each variable has exactly one definition
  $\Rightarrow$ Identity of variables and dynamic constants (values)

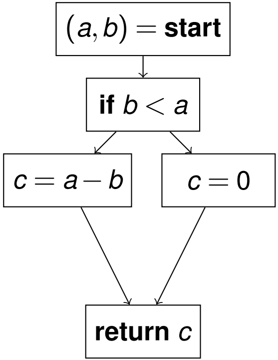
non-SSA



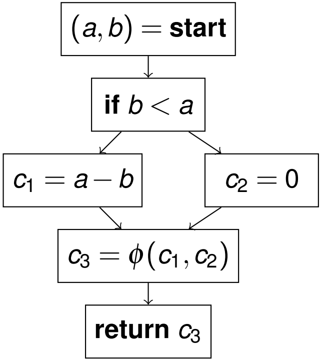
Static Single Assignment Form

- Each variable has exactly one definition
  $\Rightarrow$ Identity of variables and dynamic constants (values)
- ϕ-operations select values dependent on control flow

non-SSA



SSA

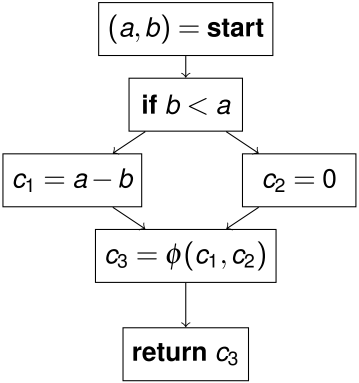


Dominance

Crucial for SSA-form programs is the concept of dominance:

Definition

ℓ_1 dominates ℓ_2 if each path from **start** to ℓ_2 goes through ℓ_1



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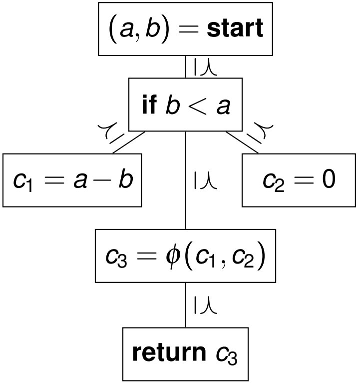
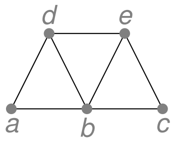


- Each node has a unique *immediate dominator*
- Thus, dominance induces a tree on the control flow graph
- Thus, dominance is also a partial order

Perfect Elimination Orders

- Restriction: To remove a node *n*, all neighbors must form a clique



elimination order
a, c, d, e, b

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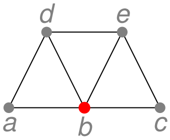

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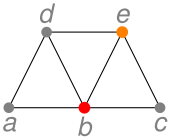

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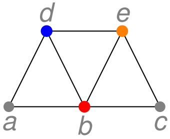

elimination order
a, c,

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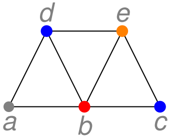

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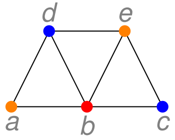
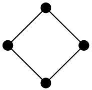


elimination order

Perfect Elimination Orders II

- Not every graph has a PEO, e.g.



- The graphs that have PEOs are exactly the class of *chordal* graphs

Perfect Elimination Orders II

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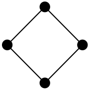


- The graphs that have PEOs are exactly the class of *chordal* graphs

Definition

A graph is chordal,
if each cycle with at least four nodes contains a chord.

Theorem

- *Chordal graphs can be optimally colored in* $O(\omega(G) \cdot |V|)$
- *Number of colors is bounded by the size* $\omega(G)$ *of the largest clique*

Outline

Main Results / Driving Force

- Interference graphs of SSA-form programs are chordal
- Dominance relation induces a PEO in the interference graph of the program
- Optimal assignment of registers in $O(\omega(G) \cdot |V|)$ without constructing the graph itself

Architecture without iteration

$\rightarrow$ Spill $\rightarrow$ Color $\rightarrow$ Coalesce $\rightarrow$ SSA destruction $\rightarrow$

Why are SSA IGs chordal? — Intuition

Program   Live Ranges

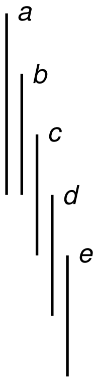
$a \leftarrow \cdots$

$b \leftarrow \cdots$

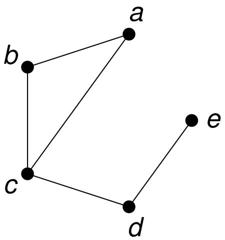
$c \leftarrow \cdots$

$d \leftarrow a + b$

$e \leftarrow c + 1$



Interference Graph



- How can we create a 4-cycle $\{a, c, d, e\}$?

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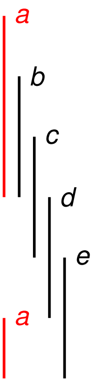
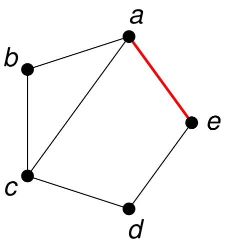
Live Ranges



Interference Graph



- How can we create a 4-cycle $\{a, c, d, e\}$?
- Redefine $a \implies$ SSA violated!

Outline

Spilling

- For any graph G: $\chi(G) \geq \omega(G)$
- For chordal graphs: $\chi(G) = \omega(G)$

Theorem

For each clique in the IG there is a label in the program where all nodes in the clique are live.

- $\chi(IG)$ is exactly determined by the size of the live sets of the labels
- Lowering the number of values live at each label to k makes the IG k-colorable
- We know in advance where values must be spilled
  $\Rightarrow$ All labels where the pressure is larger than k

Coloring

- Computing the PEO explicitly is unnecessary
- Compute liveness information
- Process basic blocks in dominance order
- After spilling $\omega(G) \leq k$

- Thus, coloring is very simple and fast $O(n)$

Coalescing

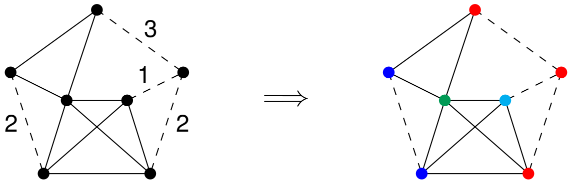
- Coalescing is optional but important

$$\rightarrow \boxed{\text{Spill}} \rightarrow \boxed{\text{Color}} \rightarrow \boxed{\text{Coalesce}} \rightarrow \boxed{\text{SSA destruction}} \rightarrow$$

- Is an isolated subordinate optimization
  - must keep register pressure below k
- Merging nodes could destroy chordality
- Information about $\chi(IG)$ would be “lost”

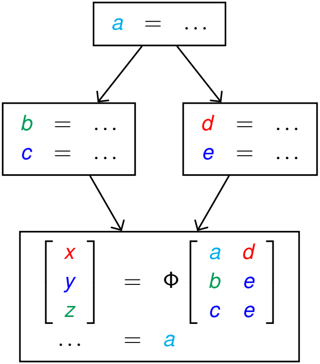
Modeling Coalescing

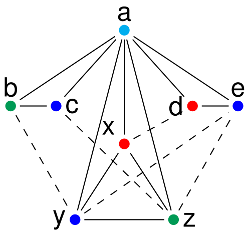
- Augment the interference graph by affinity edges representing costs of copies
- Costs are incurred if affine nodes have different colors
- Change a given coloring to reduce costs or
- find a correct coloring that minimizes the costs



Outline

Φ-Function Handling





- SSA construction inserts Φ functions
- This can be seen as live-range splitting
- Costs are modeled by affinity edges between each Φ result and its arguments

Register Constraint Handling (1)

- Register constraints correspond to a partial pre-coloring of the *IG*
- This destroys equality $\chi(G) = \omega(G)$ even for chordal graphs

- Live-range splitting can re-establish this property
- Actual problem/complexity is shifted to coalescing

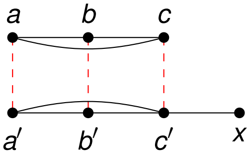
$\rightarrow$ Spill $\rightarrow$ LR splitting $\rightarrow$ Color' $\rightarrow$ Coalesce $\rightarrow$ SSA destr. $\rightarrow$

Register Constraint Handling (2)

$$
\begin{aligned}
x &= op(a, b) \\
d &= c + x
\end{aligned}
$$

- Insert a parallel copy before each constrained operation *op*

$$
\begin{aligned}
(a', b', c') &= (a, b, c) \\
x &= op(a', b') \\
d &= c' + x
\end{aligned}
$$



- This live range splitting
  - maintains $\chi(G) = \omega(G)$
  - introduces additional affinity edges

Live-Range Splitting and Colorability

- Chaitin: $\forall G \, \exists \, program \, P : IG(P) = G$
- Live-range splitting enables efficient coloring

  SSA construction $\Rightarrow$ chordal Graphs

  Constraint handling $\Rightarrow$ chordal, pre-colored Graphs

Two Similar Problems

A) Optimal Coalescing (as above)

For a given program find a coloring such that the costs for copies are minimized

B) Optimal Live-Range Splitting

For a given program find a set of split points and a coloring such that the costs for copies are minimized

- There are sets of split points such that a solution of A is a solution of B
- Characterization of such sets that are minimal is an open problem
- Splitting everywhere is a superset
  but this bloats the coalescing problems significantly

Outline

Coloring

Classical:

- Interference graph centric
- Coloring based on elimination orders
- Heuristics guide node selection

SSA:

- Interference graph is not necessary
- Coloring based on perfect elimination orders (PEO)
- Liveness and dominance information suffice

Spilling

Classical:

- No possible color for node $\longrightarrow$ spill decision
- Node weigths are used to associate spill-costs with nodes
- Restart necessary after spilling

$\Rightarrow$ Coloring enabler

SSA:

- Decisions can be based on program structure
- Only executed once

$\Rightarrow$ Program transformation reducing local register pressures to k

Coalescing

Classical:

- Merging of nodes in the IG
- Colorability of IG may suffer
- More and more complicated techniques to avoid harmful cases

SSA:

- No merging to preserve graph class and knowledge about $\chi(IG)$
- Expressed as an optimization problem to assign node pairs the same colors
- Strong connection to live-range splitting
- Handles complexity of register constraints as well

Summary

- Live-range splitting enables efficient coloring schemes
- Knowledge about $\chi(IG)$ allows sensible decoupling of subphases
- Faster allocators: no iteration, (possibly) no IG construction