# Parametric Timing Analysis for Complex Systems

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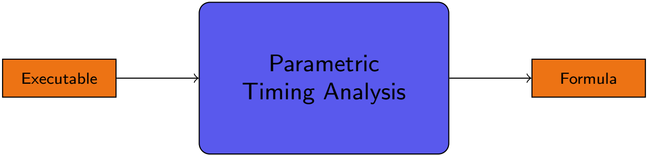
# Motivation

- timing analysis essential for hard real-time systems
- many systems depend on input parameters (operating system schedulers, etc.)
- only two possible solutions:
  1. assume upper bounds on the unknown parameters
     $\Rightarrow$ highly overapproximated WCET
  2. restart the analysis for all parameter assignments
     $\Rightarrow$ very high analysis time
- parametric timing analysis delivers timing formula instead of a numeric value

# Outline

# Structure

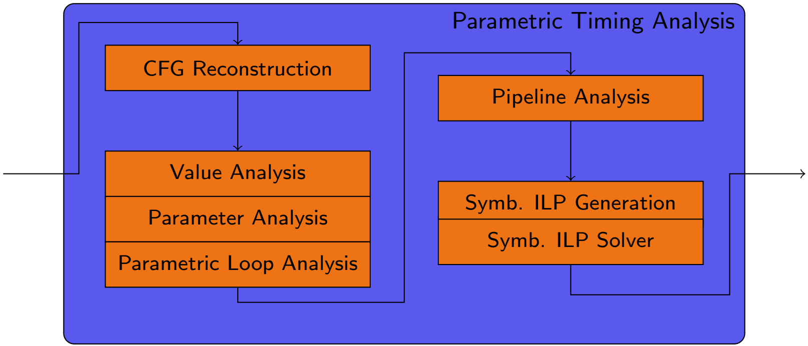



**Input** binary executable (no high-level code)

**Output** parametric timing formula

Parametric timing analysis is based on
AbsInt's *aiT Timing Analyzer*

# Internal Structure



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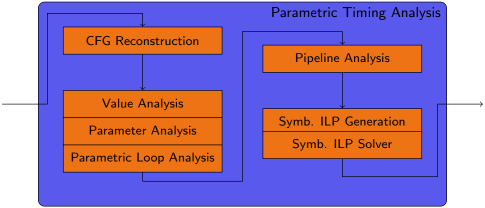


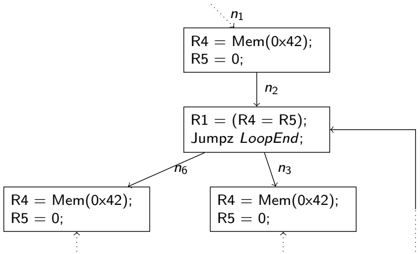

CFG Reconstruction extracts the control flow graph from the executable

# CFG Reconstruction

builds the control flow graph.

A control flow graph consists of

- basic blocks (list of instructions always entered at the first and left at the last)
- edges representing possible control flow



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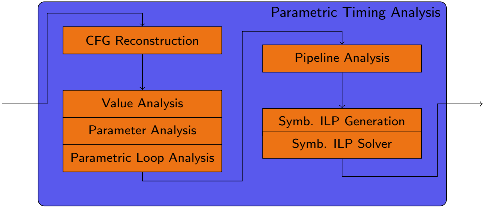



Value Analysis determines values for registers and memory accesses

Parameter Analysis determines the parameters of the analyzed program

Parametric Loop Analysis determines loop bounds and parametric loop bound expressions

# Value Analysis

determines values of registers and memory cells.

For a register x,
value analysis derives interval $[b_l, b_u]$, s.t. $b_l \leq x \leq b_u$.

Intervals are needed to

- determine addresses of memory accesses (for cache analysis),
- exclude infeasible paths,
- derive loop bounds.

# Parameter Analysis

- on source-code level, parameters and variables clearly separated
- on executable level, there are only registers and memory cells

⇒ a parameter is a register or a memory cell, the program reads from before it writes to.

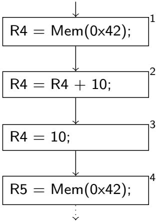
## Parameter Analysis derives

1. set of parameters,
2. parametric dependencies for all program points.

# Parameter Analysis

1. collects parametric dependencies such as: $R_x = R_y + [l_b, u_b]$
2. intervals are taken from value analysis
3. implemented as a data flow analysis

Example:




(assume memory cell Mem(0x42) has not been accessed before)

# Parameter Analysis

1. collects parametric dependencies such as: $R_x = R_y + [l_b, u_b]$
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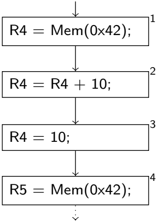
Example:

$\{Mem(0x42)\},\{\}$



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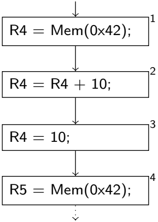


$\{Mem(0x42)\},\{(R4 = Mem(0x42) + [0,0])\}$

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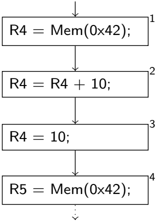


$\{Mem(0x42)\},\{\}$

$\{Mem(0x42)\},\{(R4 = Mem(0x42) + [0, 0])\}$

$\{Mem(0x42)\},\{(R4 = Mem(0x42) + [10, 10])\}$

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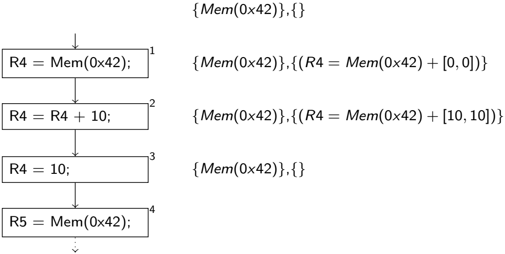

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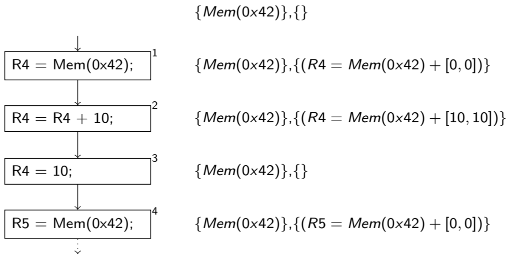


# Parametric Loop Analysis

derives symbolic loop bound expressions.

Such as a loop $L$ is bounded by $b_L$, where

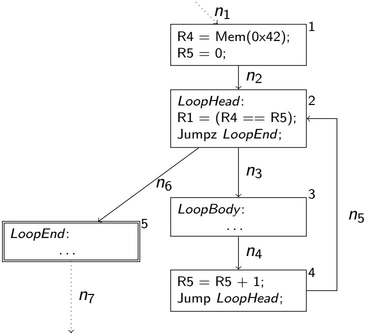
$$b_L = \text{if } Mem(0x42) < 0 \text{ then } \infty \text{ else } Mem(0x42)$$

Analysis

- acquires interval from value analysis
- acquires set of parameters from parameter analysis

# Parametric Loop Analysis

1. Collection of (potential) *loop counters*
2. Derivation of *loop invariant*
3. Evaluation of loop exits
4. Construction of loop bounds



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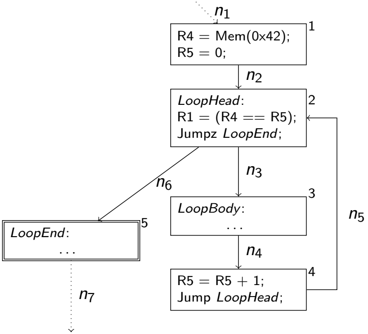


1. potential loop counters
= all register accessed within loop body
(here: $R5$)

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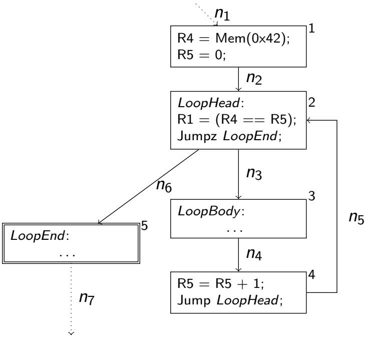


2. loop invariant
= how loop counter changes per iteration
(here: $R5 = R5 + 1$)

# Parametric Loop Analysis

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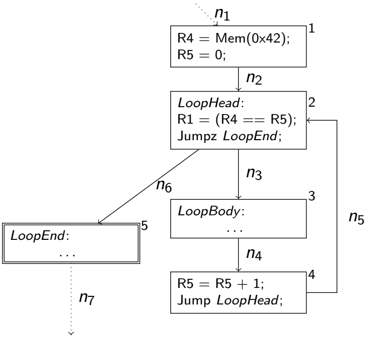


3. evaluation of loop exits (here: $R1 = (R4 == R5)$)
initial values/parameter dependencies:
$R4 = \text{Mem}(0x42), R5 = 0$

# Parametric Loop Analysis

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2. Derivation of *loop invariant*
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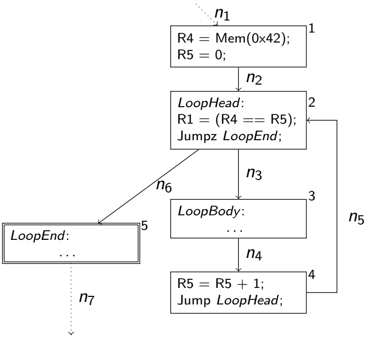
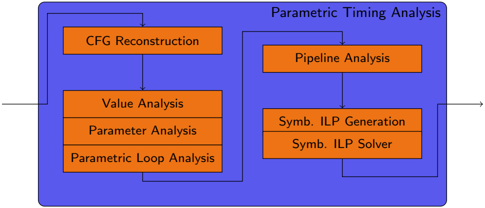



4. construction of loop bounds:

$$b_L = \text{if } Mem(0x42) < 0 \text{ then } \infty \text{ else } Mem(0x42)$$

# Internal Structure




**Pipeline Analysis** simulates the low-level behavior of the target architecture

**Symbolic ILP Generator** generates a symbolic ILP which represent the search for the longest program path

**Symbolic ILP Solver** solves the symbolic analysis ILP and returns the symbolic timing formula

# Pipeline Analysis

In modern processors, execution time depends on
caches, branch prediction, speculative execution, ...

Thus, pipeline analysis

- simulates processors low level behavior,
- determines WCETs for all basic blocks.

# Parametric Path Analysis

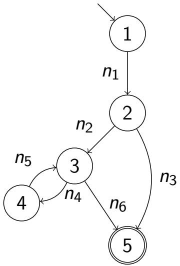
computes WCET by searching longest path.

Analysis consists of two steps:

1. creating ILP with parameters
2. solving symbolic ILP to produce symbolic timing formula

# Parametric Path Analysis - IPET

Searching for path with highest execution time by using
Implicit Path Enumeration Technique (IPET)




variables $n_i$ denote how often edge $i$ is traversed

# Parametric Path Analysis - IPET

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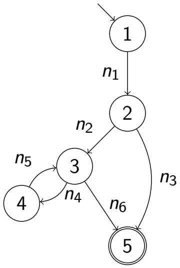


$n_1 = 1;$

first node is entered exactly once

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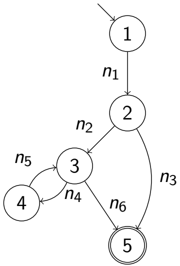
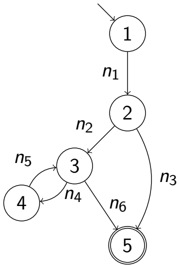



$$n_1 = 1;$$
$$n_1 = n_2 + n_3;$$
$$n_2 + n_5 = n_4 + n_6;$$
$$n_4 = n_5;$$

sum of successors traversals equals sum of predecessor traversals

# Parametric Path Analysis - IPET

Searching for path with highest execution time by using Implicit Path Enumeration Technique (IPET)

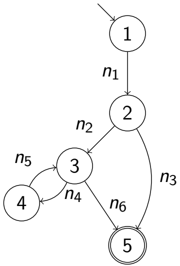



$$
\begin{aligned}
n_1 &= 1; \\
n_1 &= n_2 + n_3; \\
n_2 + n_5 &= n_4 + n_6; \\
n_4 &= n_5; \\
n_4 &<= b_L n_2;
\end{aligned}
$$

loop $L$ is executed $b_L$ times as often as it is entered
($b_L$ represents loop bound expression)

# Parametric Path Analysis - IPET

Searching for path with highest execution time by using Implicit Path Enumeration Technique (IPET)

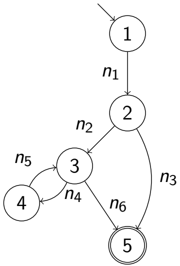



$$
\begin{aligned}
n_1 &= 1; \\
n_1 &= n_2 + n_3; \\
n_2 + n_5 &= n_4 + n_6; \\
n_4 &= n_5; \\
n_4 &<= b_L n_2; \\
n_3 + n_6 &= 1;
\end{aligned}
$$

last node is entered exactly once

# Parametric Path Analysis - IPET

Searching for path with highest execution time by using Implicit Path Enumeration Technique (IPET)




$$n_1 = 1;$$
$$n_1 = n_2 + n_3;$$
$$n_2 + n_5 = n_4 + n_6;$$
$$n_4 = n_5;$$
$$n_4 <= b_L n_2;$$
$$n_3 + n_6 = 1;$$

$$\max : \sum_i \left( \sum_{\forall j : n_j \text{ enters } B_i} c_i n_j \right)$$

objective function: maximize execution time by maximizing $c_i n_j$
($c_i$ = WCET of basic block $n_J$ enters)

# Parametric Path Analysis - Non-linear IPET constraints

relative loop constraint

$$n_4 <= b_L n_2;$$

is non-linear and has to be replaced by absolute constraint

$$n_4 <= c \cdot b_L;$$

by bounding variable $n_2$ by a constant $c$.

Note: all variables $n_i$ can be bounded if all loop bounds for all loops exist

# Parametric Path Analysis - Symbolic ILP

Symbolic ILP with parameters representing loop bound expressions.

Using PIP[1] to solve symbolic ILPs

PIP uses

- symbolic simplex, and
- symbolic cutting plane algorithm.

Note:

- PIP is usually bottleneck of the analysis
- number of parameters only bounded by performance of PIP

[1] http://www.piplib.org

# Short Note on the Timing Formulae

PIP outputs QUAST-file which is hardly human-readable

⇒ one additional step needed to:

1. pretty-print timing formula
2. evaluate symbolic loop bound expressions, and
3. evaluate timing formula

# Example of a Timing Formulae

```
(if #[ 0 1 -1]
 (list                          (list
  #[ 1 -376 -274]                #[ 1 -428 -222]
  #[ 0 0 1]                      #[ 0 1 0]
     ...                            ...
  #[ 0 0 0]                      #[ 0 0 0]
  #[ 0 0 0]                      #[ 0 0 0]
     ...                            ...
  #[ 0 0 1]                      #[ 0 1 0]
 )                              )
)
```

# Example of a Timing Formulae

first row only artificial – can be neglected

last row denotes a constant

$$\begin{bmatrix} 0 & 7 & 3 \end{bmatrix}$$

middle row(s) denotes the parameter(s)

$$= 7p + 3$$

# Example of a Timing Formulae

```
(if #[ 0 1 -1]
 (list
  #[ 1 -376 -274]
  #[ 0 0 1]
     ...
  #[ 0 0 0]
  #[ 0 0 0]
     ...
  #[ 0 0 1]
 )
 (list
  #[ 1 -428 -222]
  #[ 0 1 0]
     ...
  #[ 0 0 0]
  #[ 0 0 0]
     ...
  #[ 0 1 0]
 )
)
```

# Example of a Timing Formulae

if $(p - 1 \geq 0)$

```
(if #[ 0 1 -1]
```

then $(376p + 274)$

```
#[ 1 -376 -274]
#[ 0 0 1]
    ...
#[ 0 0 0]
#[ 0 0 0]
    ...
#[ 0 0 1]
)
```

else $(428p + 222)$

```
#[ 1 -428 -222]
#[ 0 1 0]
    ...
#[ 0 0 0]
#[ 0 0 0]
    ...
#[ 0 1 0]
)
)
```

# Precision

Parametric analysis (PA) has less information than numeric analysis (NA)
$\Rightarrow$ PA at most as precise as NA

Two main sources for a worse precision:

1. absolute loop bounds, and
2. path-exclusion.

# Precision - Absolute Loop Bounds

1. absolute loop bounds:

   absolute loop bounds ($n_2 <= b_L n_1;$) are less precise than relative ones ($n_2 <= c \cdot b_L;$)

   $c$ is only a bound, and thus can overapproximate true loop entry count

2. path-exclusion:

   numeric analysis knows the actuals values during analysis

   $\Rightarrow$ numeric analysis can exclude some paths which the parametric can not

# Testsetting

- Parametric timing analysis for PowerPC 565 and 755
- examples from Mälardalen WCET benchmark suite
  - Insertion sort
  - Matrix Multiplication
  - Square Root Computation by Taylor Series
  - Cyclic Redundancy Check (2 parameters)
- performed on: Intel Core Duo 1,66 Mhz, 1024 MB RAM
- compiled with: gcc-cross-compiler

# Testcases

We compared parametric analysis (PA) to numeric analysis (NA)

- **PA** compute timing formula once and instantiate it
- **NA** compute one WCET bound for each value-assignment

# Insertion Sort

- one normal parametric loop (initializing an array)
- one nested parametric loop that sorts the values (according to insertion-sort)
- parametric in the size of the array

| n | NA | PA | Diff. (%) |
|---|---|---|---|
| 0 | 1 494 | 1 798 | 20.3 |
| 1 | 1 910 | 2 086 | 9.1 |
| 10 | 118 411 | 121 579 | 2.7 |
| 100 | 10 788 631 | 10 791 799 | $<0.1$ |

$$Time(n) = \begin{cases} 1798 & \text{if } n < 1 \\ 2086 & \text{if } n = 1 \\ 1067n^2 + 1188n + 2999 & \text{otherwise} \end{cases}$$

# Matrix Multiplication

- one nested parametric loop initializing the two matrices
- nested parametric loops of depth 3 to multiply matrices (naive approach)
- parametric in the dimension of the matrices ($n \times n$)

| n | NA | PA | Diff. (%) |
|---|---|---|---|
| 0 | 2 915 | 3 046 | 4.5 |
| 1 | 5 244 | 8 371 | 59.6 |
| 10 | 745 156 | 890 884 | 19.6 |
| 100 | 669 888 316 | 683 246 374 | 2.0 |

$$Time(n) = \begin{cases} 3046 & \text{if } n < 1 \\ 2431n^3 + 2003n^2 \\ \quad +663n + 3274 & \text{otherwise} \end{cases}$$

# Square Root Computation

- computes square root using Taylor series
- one parametric loop
- parameter adjusts iteration depth/precision

| n | NA | PA | Diff. (%) |
|---|---|---|---|
| 0 | 208 | 208 | 0 |
| 1 | 208 | 208 | 0 |
| 10 | 3331 | 3331 | 0 |
| 100 | 34561 | 34561 | 0 |

$$Time(n) = \begin{cases} 208 & \text{if } n < 1 \\ 347(n-1) + 208 & \text{otherwise} \end{cases}$$

# Cyclic Redundancy Check

- 8bit cyclic redundancy check
- two parametric loops (not nested)
- two different parameters:
  - $n =$ the length of the input stream
  - $a$ restricts size of input alphabet

# Results - Cyclic Redundancy Check

| n | a | NA | PA | Diff. (%) |
|---|---|---|---|---|
| 1 | 64 | 50545 | 50545 | 0 |
| 1 | 256 | 203377 | 203377 | 0 |
| 10 | 64 | 51904 | 51904 | 0 |
| 10 | 256 | 204736 | 204736 | 0 |
| 100 | 64 | 65494 | 65494 | 0 |
| 100 | 256 | 218326 | 218326 | 0 |

$$Time(n,a) = \begin{cases} 397 & \text{if } n < 1 \wedge a < 1 \\ 796(a-1) + 397 & \text{if } n < 1 \wedge a \geq 1 \\ 151(n-1) + 397 & \text{if } n \geq 1 \wedge a < 1 \\ 151(n-1) + 796(a-1) + 397 & \\ & \text{if } n \geq 1 \wedge a \geq 1 \end{cases}$$

# Conclusions

Parametric timing analysis

- derives symbolic WCET formula automatically
  - identifies the parameters/computes parametric dependencies
  - derives symbolic loop bound expressions
  - creates and solves symbolic ILP to determine WCET formula
- is implemented as a prototype for PowerPC 565/755

Results of the parametric analysis:

- are usually as precise as numeric results for non-nested loops,
- converge to the numeric results as the values increase for nested loops

Usually, symbolic ILP solver is bottleneck of the analysis.

**Thanks for your attention!**